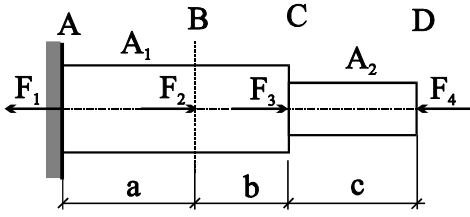


## AKSIJALNO OPTEREĆENI NOSAČI

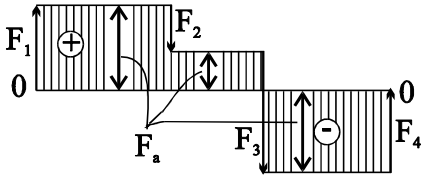
$$\sigma = \frac{F_a}{A}, \quad \Delta l = \frac{F_a l}{AE}, \quad \Delta l_t = l \alpha \Delta t,$$

$$\text{dimenzionisanje } |\sigma|_{\max} \leq \sigma_d$$

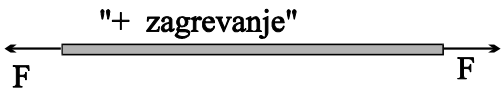


$$\Delta l_{A-D} = \Delta l_{A-B} + \Delta l_{B-C} + \Delta l_{C-D}$$

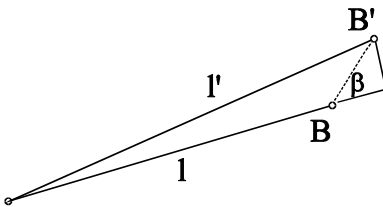
$$\Delta l_{A-B} = \frac{F_1 a}{A_1 E}, \quad \Delta l_{B-C} = \frac{(F_1 - F_2) b}{A_1 E}, \quad \Delta l_{C-D} = -\frac{F_4 c}{A_2 E}$$



$$\sigma_{A-B} = \frac{F_1}{A_1}, \quad \sigma_{B-C} = \frac{(F_1 - F_2)}{A_1}, \quad \sigma_{C-D} = -\frac{F_4}{A_2}$$



$$\Delta l = \frac{Fl}{AE} + l \alpha \Delta t$$



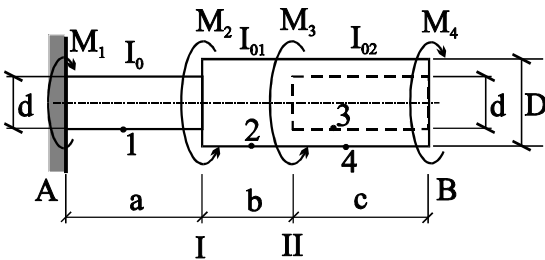
l'-dužina elastičnog šrtapa nakon deformacije  
 l-dužina elastičnog šrtapa pre deformacije  
 $\Delta l \approx \overline{BB'} \cos \beta$

## UVIJANJE

$$\tau = \frac{M_u}{I_0} \rho, \quad \rho_{\max} = \text{poluprečnik},$$

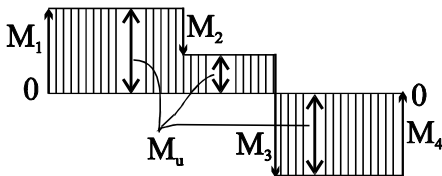
$$\theta = \frac{M_u l}{GI_0}$$

$$\text{dimenzionisanje } \tau_{\max} \leq \tau_d$$



$$\tau_1 = \frac{M_1 d}{I_0}, \quad \tau_2 = \frac{(M_1 - M_2) D}{I_{01}}, \quad \tau_4 = \frac{M_4 D}{I_{02}}$$

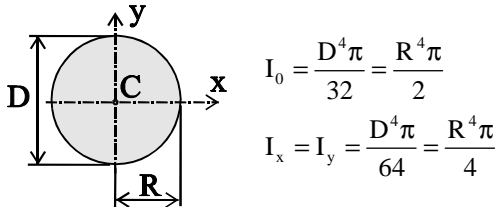
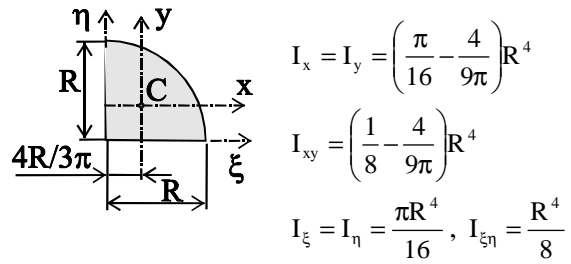
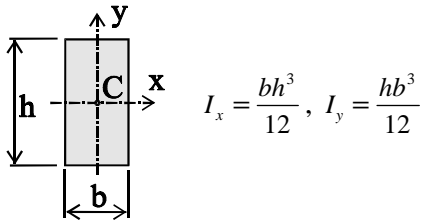
$$I_0 = \frac{d^4 \pi}{32}, \quad I_{02} = \frac{\pi}{32} (D^4 - d^4)$$



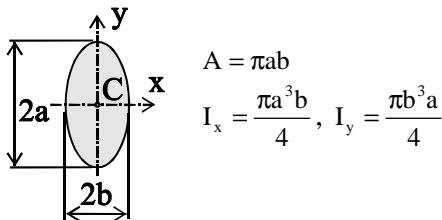
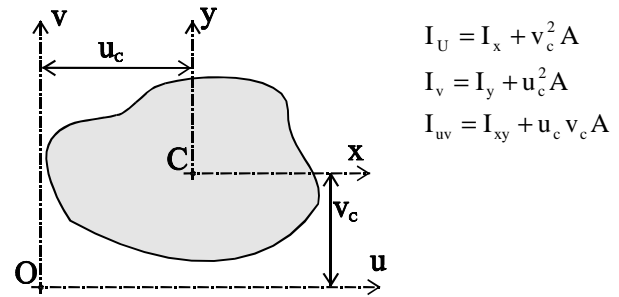
$$\theta_{A-B} = \theta_{A-I} + \theta_{I-II} + \theta_{II-B}$$

$$\theta_{A-I} = \frac{M_1 a}{GI_0}, \quad \theta_{I-II} = \frac{(M_1 - M_2) b}{GI_{01}}, \quad \theta_{II-B} = -\frac{M_4 c}{GI_{02}}$$

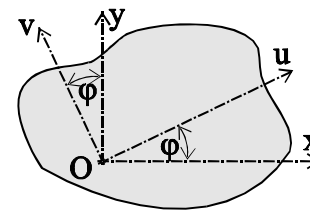
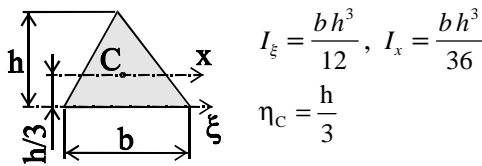
MOMENTI INERCIJE ELEMENTARNIH POVRŠINA



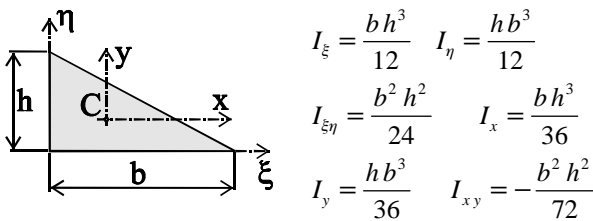
VEZA IZMEĐU MOMENATA INERCIJE ZA PARALELNE OSE – ŠTAJNEROVI OBRASCI



MOMENTI INERCIJE ZA ZAKRENUT KOORDINATNI SISTEM



$I_u = I_x \cos^2 \phi + I_y \sin^2 \phi - I_{xy} \sin 2\phi$



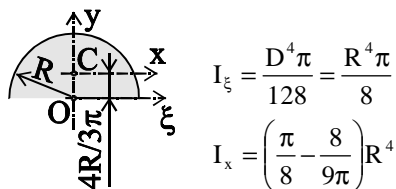
$I_{uv} = \frac{I_x - I_y}{2} \sin 2\phi + I_{xy} \cos 2\phi$

GLAVNE OSE – GLAVNI MOMENTI INERCIJE u i v postaju glavne ose 1 i 2 (odnosno 2 i 1) za  $\phi = \alpha$

$\text{tg } 2\alpha = -\frac{2I_{xy}}{I_x - I_y}$

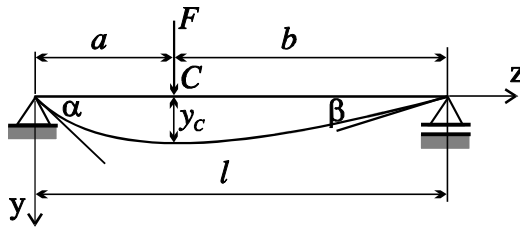
$I_{1,2} = \frac{I_x + I_y}{2} \pm \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$\left. \frac{d^2 I_u}{d\phi^2} \right|_{\phi=\alpha} = 2(I_y - I_x) \cos 2\alpha + 4I_{xy} \sin 2\alpha$



INVARIJANTE MOMENATA INERCIJE

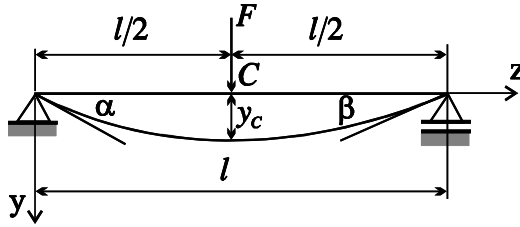
$I_1 + I_2 = I_x + I_y, \quad I_1 I_2 = I_x I_y - I_{xy}^2$



$$y = \frac{Fl^3}{6EI} \left\{ \frac{bz}{l} \left[ 1 - \left( \frac{b}{l} \right)^2 - \left( \frac{z}{l} \right)^2 \right] + \left\| \left( \frac{z-a}{l} \right)^3 \right\| \right\}$$

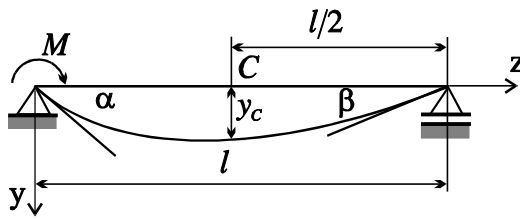
$$y(a) = y_c = \frac{Fl^3}{3EI} \left( \frac{a}{l} \right)^2 \left( \frac{b}{l} \right)$$

$$\alpha = \frac{Fl^2}{6EI} \frac{ab}{l} \left( 1 + \frac{b}{l} \right) \quad \beta = \frac{Fl^2}{6EI} \frac{ab}{l} \left( 1 + \frac{a}{l} \right)$$



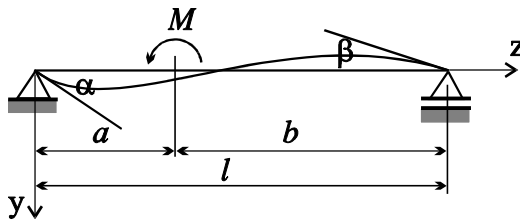
$$y = \frac{Fl^3}{12EI} \left\{ \frac{z}{l} \left[ \frac{3}{4} - \left( \frac{z}{l} \right)^2 \right] + \left\| \frac{l}{4} \left( 2 \frac{z}{l} - 1 \right)^3 \right\| \right\}$$

$$y\left(\frac{l}{2}\right) = y_c = y_{\max} = \frac{Fl^3}{48EI} \quad \alpha = \beta = \frac{Fl^2}{16EI}$$



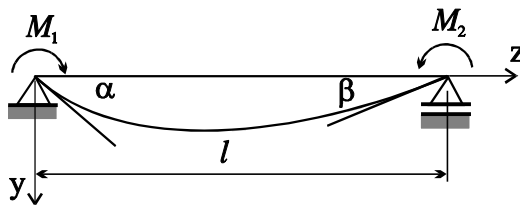
$$y = \frac{Ml^2}{6EI} \frac{z}{l} \left[ 2 - 3 \left( \frac{z}{l} \right)^2 + \left( \frac{z}{l} \right)^4 \right]$$

$$y\left(\frac{l}{2}\right) = y_c = \frac{Ml^2}{16EI} \quad \alpha = \frac{Ml}{3EI} \quad \beta = \frac{Ml}{6EI}$$



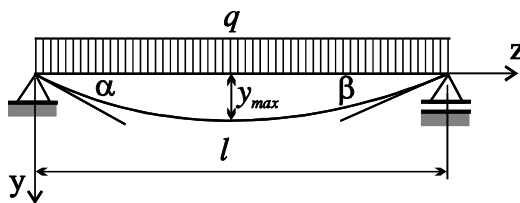
$$y = \frac{Ml^2}{6EI} \left\{ \frac{z}{l} \left[ 1 - 3 \left( \frac{b}{l} \right)^2 - \left( \frac{z}{l} \right)^2 \right] + \left\| 3 \left( \frac{z-a}{l} \right)^2 \right\| \right\}$$

$$\alpha = \frac{Ml}{6EI} \left[ 1 - 3 \left( \frac{b}{l} \right)^2 \right] \quad \beta = \frac{Ml}{6EI} \left[ 1 - 3 \left( \frac{a}{l} \right)^2 \right]$$



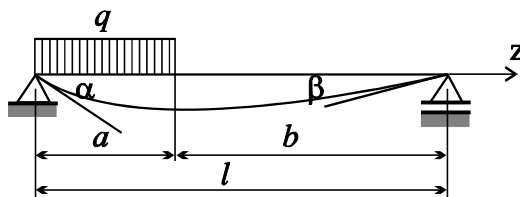
$$y = \frac{l^2}{6EI} \frac{z}{l} \left( 1 - \frac{z}{l} \right) \left[ M_1 \left( 2 - \frac{z}{l} \right) + M_2 \left( 1 + \frac{z}{l} \right) \right]$$

$$\alpha = \frac{l}{6EI} (2M_1 + M_2) \quad \beta = \frac{l}{6EI} (M_1 + 2M_2)$$



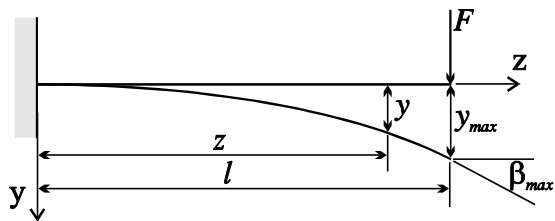
$$y = \frac{ql^4}{24EI} \left[ \frac{z}{l} - 2 \left( \frac{z}{l} \right)^3 + \left( \frac{z}{l} \right)^4 \right]$$

$$y\left(\frac{l}{2}\right) = y_{\max} = \frac{5ql^4}{384EI} \quad \alpha = \beta = \frac{ql^3}{24EI}$$



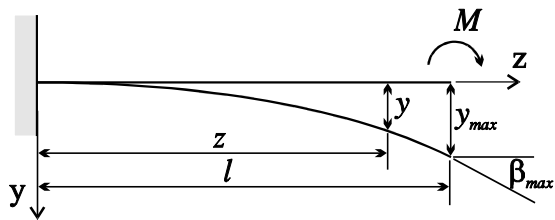
$$y = \frac{ql^4}{24EI} \left\{ \frac{z}{l} \left( 2 - \frac{a}{l} \right)^2 \left( \frac{a}{l} \right)^2 - \frac{z}{l} \left[ 2 \left( 2 - \frac{a}{l} \right) \frac{a}{l} \left( \frac{z}{l} \right)^2 - \left( \frac{z}{l} \right)^3 \right] + \left\| \left( \frac{z-a}{l} \right)^4 \right\| \right\}$$

$$\alpha = \frac{ql^3}{24EI} \left( \frac{a}{l} \right)^2 \left( 2 - \frac{a}{l} \right)^2 \quad \beta = \frac{ql^3}{24EI} \left( \frac{a}{l} \right)^2 \left[ 2 - \left( \frac{a}{l} \right)^2 \right]$$



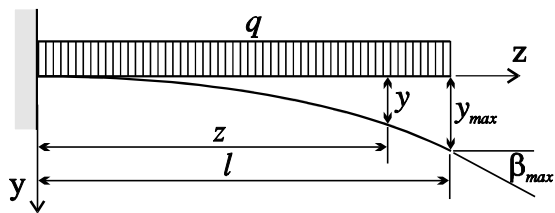
$$y = \frac{Fl^3}{6EI} \left(\frac{z}{l}\right)^2 \left(3 - \frac{z}{l}\right) \quad y(l) = y_{\max} = \frac{Fl^3}{3EI}$$

$$\beta_{\max} = \frac{Fl^2}{2EI}$$



$$y = \frac{Ml^2}{2EI} \left(\frac{z}{l}\right)^2 \quad y(l) = y_{\max} = \frac{Ml^2}{2EI}$$

$$\beta_{\max} = \frac{Ml}{EI}$$



$$y = \frac{ql^4}{24EI} \left(\frac{z}{l}\right)^2 \left[6 - 4\left(\frac{z}{l}\right) + \left(\frac{z}{l}\right)^2\right] \quad y(l) = y_{\max} = \frac{ql^4}{8EI}$$

$$\beta_{\max} = \frac{ql^3}{6EI}$$

### normalni i tangencijalni naponi pri savijanju

$$|\sigma_c| = \frac{|M_s|}{I_x} |y_c|, \quad |\tau_c| = \frac{|F_T|}{I_x} \frac{S_c}{\xi}, \quad S_c = S_x^{(A_c)} = A_c \cdot y_t$$

### glavni naponi pri savijanju

$$\sigma_1 = \frac{\sigma}{2} + \frac{1}{2} \sqrt{\sigma^2 + 4\tau^2} \quad \sigma_2 = \frac{\sigma}{2} - \frac{1}{2} \sqrt{\sigma^2 + 4\tau^2}$$

### x dimenzionisanje pri savijanju

$$|\sigma_{\max}| = \frac{|M_{S\max}|}{I_x} |y_{\max}| = \frac{|M_{S\max}|}{W_x} \leq \sigma_d \quad W_x = \frac{I_x}{|y_{\max}|}$$

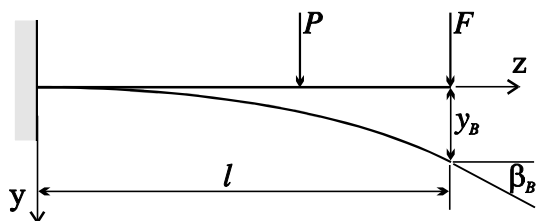
$$\Rightarrow W_x \geq \frac{|M_{S\max}|}{\sigma_d}$$

diferencijalna jednačina elastične linije  $EIy'' = -M$

težište složene površine

$$u_T = \frac{\sum u_i A_i}{\sum A_i}, \quad v_T = \frac{\sum v_i A_i}{\sum A_i} \quad \text{gde je } \sum A_i = A$$

statički moment složene površine za osu x  $S_x = \sum y_i A_i = y_T A$



### superponiranje deformacija

$$y_B = y_B^{(F)} + y_B^{(P)} \quad \text{odnosno } y_B = \sum y_{Bi}$$

$$\beta_B = \beta_B^{(F)} + \beta_B^{(P)} \quad \text{odnosno } \beta_B = \sum \beta_{Bi}$$

$$M_{k-1} l_k + 2M_k \left( l_k + l_{k+1} \frac{I_k}{I_{k+1}} \right) + M_{k+1} l_{k+1} \frac{I_k}{I_{k+1}} = -6EI_k \sum_K (\alpha_s + \beta_s) \quad \text{KLAPEJRONOVA JEDNAČINA}$$