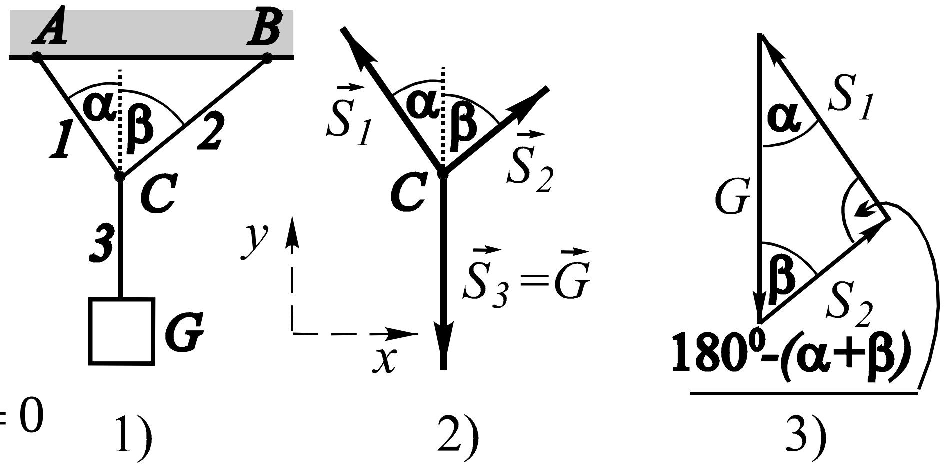
Primer 4.3

Poznate veličine: α, β, G

Odrediti: $\vec{S}_1$, $\vec{S}_2$



Prvi način: uslovi ravnoteže, **Sl.2)**

$$\sum X_i = 0 \Rightarrow -\boldsymbol{S_1}\sin\alpha + \boldsymbol{S_2}\sin\beta = 0$$

$$\sum Y_i = 0 \Rightarrow \boldsymbol{S_1}\cos\alpha + \boldsymbol{S_2}\cos\beta - G = 0$$

Množenjem prve od ovih jednačina sa $\cos\alpha$, druge sa $\sin\alpha$, pa sabiranjem

$$\Rightarrow \quad \boldsymbol{S_2}(\sin\alpha\cos\beta + \cos\alpha\sin\beta) - G\sin\alpha = 0$$

$$\Rightarrow \quad \boldsymbol{S_2} = \frac{G\sin\alpha}{\sin(\alpha+\beta)}, \qquad \boldsymbol{S_1} = \frac{G\sin\beta}{\sin(\alpha+\beta)}$$

$$\sin[180^0 - (\alpha+\beta)] = \sin(\alpha+\beta)$$

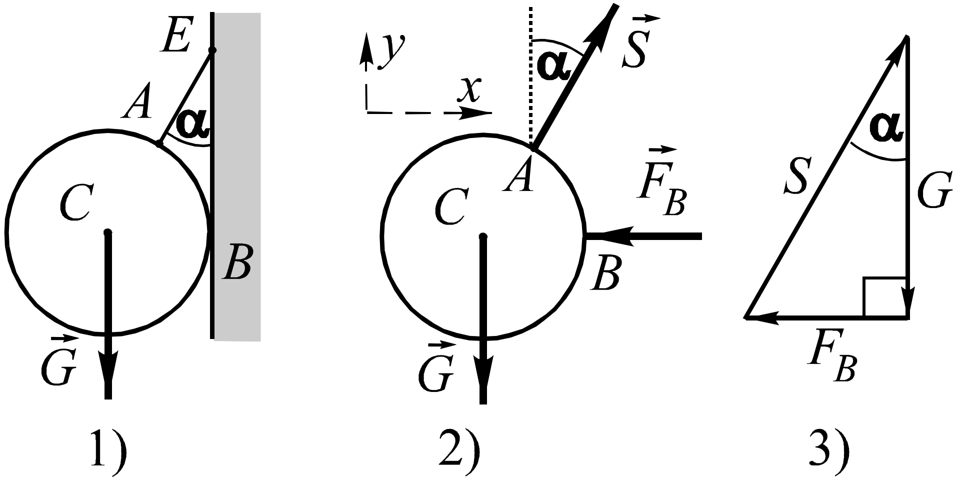
Drugi način: poligon sila, **Sl.3)**

$$\frac{G}{\sin[180^0 - (\alpha+\beta)]} = \frac{\boldsymbol{S_1}}{\sin\beta} = \frac{\boldsymbol{S_2}}{\sin\alpha} \Rightarrow$$

Primer 4.4

Poznate veličine: α, G

Odrediti: $\vec{S}$, $\vec{F}_B$



***Prvi način: poligon sila,* Sl.3)**

$$\tan\alpha = \frac{\boldsymbol{F_B}}{G} \quad \Rightarrow \quad \boldsymbol{F_B} = G\tan\alpha$$

$$\cos\alpha = \frac{G}{\boldsymbol{S}} \quad \Rightarrow \quad \boldsymbol{S} = \frac{G}{\cos\alpha}$$

Drugi način: uslovi ravnoteže

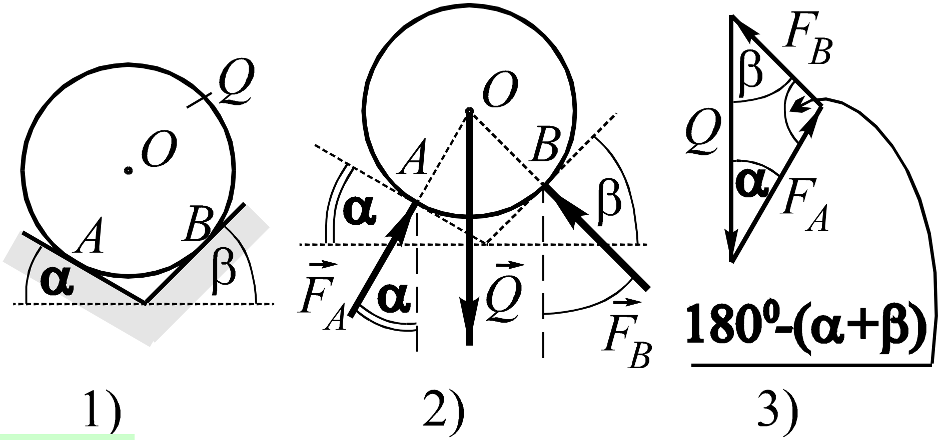
$$\sum Y_i = 0 \Rightarrow \boldsymbol{S}\cos\alpha - G = 0 \quad \Rightarrow \quad \boldsymbol{S} = \frac{G}{\cos\alpha}$$

$$\sum X_i = S\sin\alpha - \boldsymbol{F_B} = 0 \quad \Rightarrow \quad \boldsymbol{F_B} = G\tan\alpha$$

Primer 4.5

Poznate veličine: α, β, Q

Odrediti: $\vec{F}_A$, $\vec{F}_B$



***Prvi način: poligon sila,* Sl.3)**

Primena sinusne teoreme na trougao sila

$$\frac{Q}{\sin\left[180^0-(\alpha+\beta)\right]}=\frac{\boldsymbol{F_A}}{\sin\beta}=\frac{\boldsymbol{F_B}}{\sin\alpha}$$

$$\Rightarrow \boldsymbol{F_A}=\frac{Q\sin\beta}{\sin(\alpha+\beta)},\quad \boldsymbol{F_B}=\frac{Q\sin\alpha}{\sin(\alpha+\beta)}$$

Drugi način: uslovi ravnoteže

$$\sum X_i=0\Rightarrow \boldsymbol{F}_A\sin\alpha-\boldsymbol{F}_B\sin\beta=0 \quad \left|\cdot\cos\beta\right.$$

$$\sum Y_i=0\Rightarrow \boldsymbol{F}_A\cos\alpha+\boldsymbol{F}_B\cos\beta-Q=0 \quad \left|\cdot\sin\beta\right.$$

Nakon naznačenog množenja ovih jednačina pa sabiranja i korišćenja identiteta

$$\sin\alpha\cos\beta+\cos\alpha\sin\beta=\sin(\alpha+\beta)$$

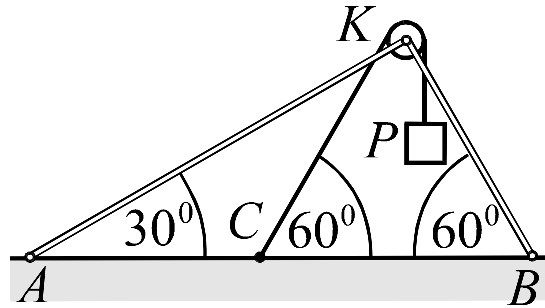
dobija se $\quad \boldsymbol{F}_A(\sin\alpha\cos\beta+\cos\alpha\sin\beta)-Q\sin\beta=0\Rightarrow \boldsymbol{F}_A=\dfrac{Q\sin\beta}{\sin(\alpha+\beta)}$

zatim se na osnovu prve jednačine dobija $\quad \boldsymbol{F}_B=\dfrac{\boldsymbol{F}_A\sin\alpha}{\sin\beta}=\dfrac{Q\sin\alpha}{\sin(\alpha+\beta)}$

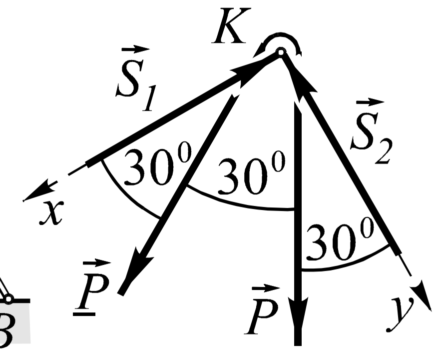
Primer 4.6

Poznata veličina: P

Odrediti: S_1, S_2



1)



2)

Uslovi ravnoteže, **Sl. 2)**

$$\sum X_i = -\boldsymbol{S_1} + P\cos 30^0 + P\cos 60^0 = 0$$

$$\sum Y_i = -\boldsymbol{S_2} + P\cos 30^0 + P\cos 60^0 = 0 \qquad \Rightarrow \qquad \boldsymbol{S_1} = +\frac{\sqrt{3}+1}{2}P, \quad \boldsymbol{S_2} = +\frac{\sqrt{3}+1}{2}P$$

Za reakcije lakih štapova usvojeni su smerovi koji su u skladu sa pretpostavkom da su oba laka štapa pritisnuta.

Predznaci “+” u dobijenim rešenjima ukazuju na to da su laki štapovi opterećeni baš kao što je i pretpostavljeno, to znači da su oba pritisnuta.

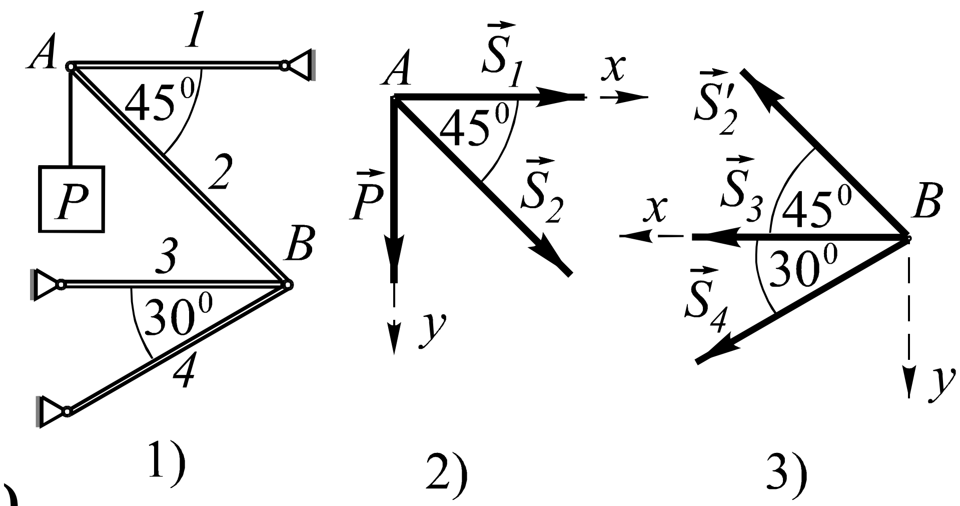
Intenzitet sile u lakom štapu uvek je jednak vrednosti koja stoji iza predznaka.

Da su drugačije usvojeni smerovi za sile u lakim štapovima (što bi odgovaralo pretpostavci da su laki štapovi zategnuti) rezultati bi se u odnosu na dobijene razlikovali samo u predznacima, ali bi zaključci, u vezi intenziteta sila i karaktera opterećenja, ostali isti.

Primer 4.7

Poznata veličina: P

Odrediti: S_1, S_2, S_3, S_4



***Uslovi ravnoteže tačke A*, Sl. 2)**

$$\sum Y_i = \boldsymbol{S_2} \sin 45^0 + P = 0 \quad \Rightarrow \quad \boldsymbol{S_2} = -\sqrt{2}P$$

$$\sum X_i = S_2 \cos 45^0 + \boldsymbol{S_1} = 0 \quad \Rightarrow \quad \boldsymbol{S_1} = P$$

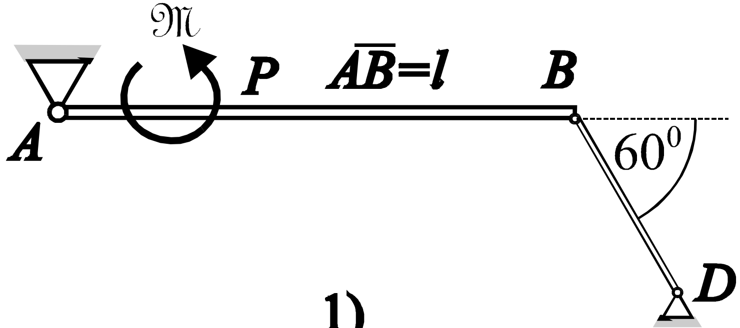
***Uslovi ravnoteže tačke B*, Sl. 3)**

$$\sum Y_i = \boldsymbol{S_4} \sin 30^0 - S_2 \sin 45^0 = 0 \Rightarrow \boldsymbol{S_4} = -2P$$

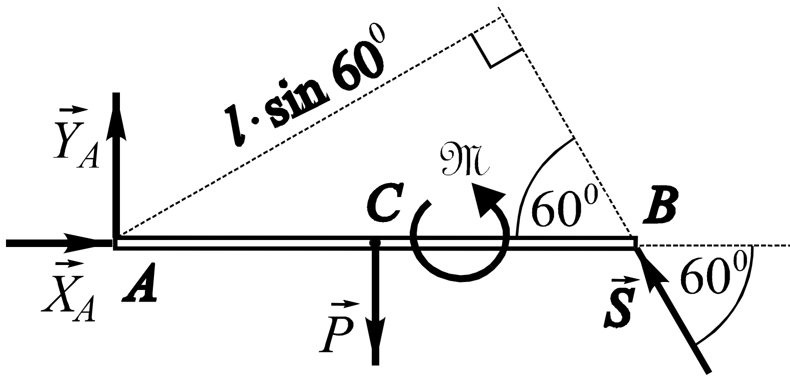
$$\sum X_i = S_2 \cos 45^0 + \boldsymbol{S_3} + S_4 \cos 30^0 = 0 \quad \Rightarrow \quad \boldsymbol{S_3} = \left(1 + \sqrt{3}\right)P$$

Primer 6.2

Homogeni štap *AB* težine *P*, dužine *l*, nalazi se u ravnoteži u horizontalnom položaju (Sl.1). Na štap dejstvuje spreg momenta $\mathfrak{M}$ smera datog na slici. Štap je u tački *A* zglobno vezan a u tački *B* se podupire na laki štap *BD* koji sa horizontalom gradi ugao od 60^0. Odrediti reakcije veza u zavisnosti od poznatih veličina $\mathfrak{M}$, *P* i *l*.



1)



2)

$$\sum M_{Ai} = -P\cdot\frac{l}{2} + S\cdot l\sin 60^0 + \mathfrak{M} = 0 \quad \Rightarrow$$

$$S = \frac{\sqrt{3}}{3}P - \frac{2\sqrt{3}}{3}\frac{\mathfrak{M}}{l}$$

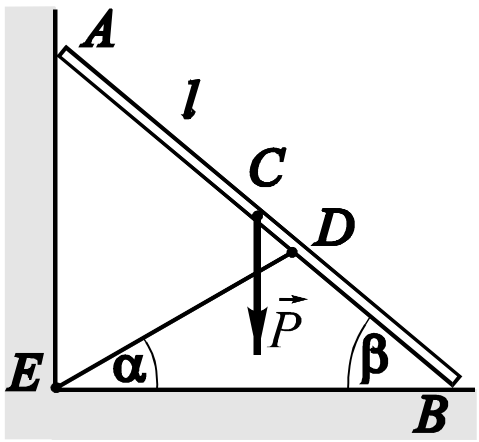
$$\sum X_i = X_A - S\cos 60^0 = 0 \quad \Rightarrow$$

$$X_A = \frac{\sqrt{3}}{6}P - \frac{\sqrt{3}}{3}\frac{\mathfrak{M}}{l}$$

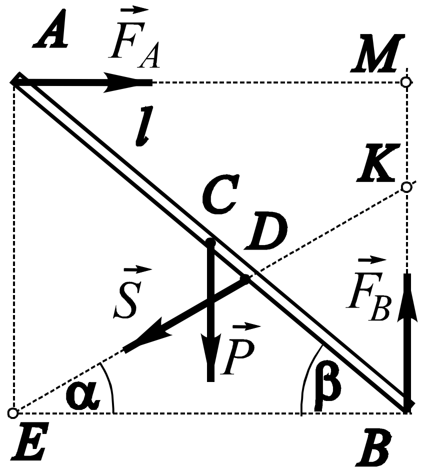
$$\sum Y_i = Y_A - P + S\sin 60^0 = 0 \quad \Rightarrow$$

$$Y_A = \frac{1}{2}P + \frac{\mathfrak{M}}{l}$$

Primer 6.3



1)



2)

Homogeni štap AB težine P, dužine l, koji sa horizontalom gradi ugao β, naslanja se u tački A na gladak vertikalni zid a u tački B na gladak horizontalni pod (Sl.1). Za tačku D štapa vezano je uže ED koje sa horizontalom gradi ugao α, kako je to na slici prikazano. Odrediti sve reakcije veza u zavisnosti od poznatih veličina α, β, P i l.

$$\sum M_{Ki} = P \cdot \frac{l}{2}\cos\beta - F_A \cdot \overline{MK} = 0$$

$$\Rightarrow F_A = \frac{Pl\cos\beta}{2\overline{MK}} = \frac{P\cos\alpha\cos\beta}{2\sin(\beta-\alpha)}$$

$$\overline{MK} = \overline{MB} - \overline{KB} = l\sin\beta - l\cos\beta\tan\alpha \quad \Rightarrow$$

$$\overline{MK} = \frac{l\sin\beta\cos\alpha - l\cos\beta\sin\alpha}{\cos\alpha} = \frac{l\sin(\beta-\alpha)}{\cos\alpha}$$

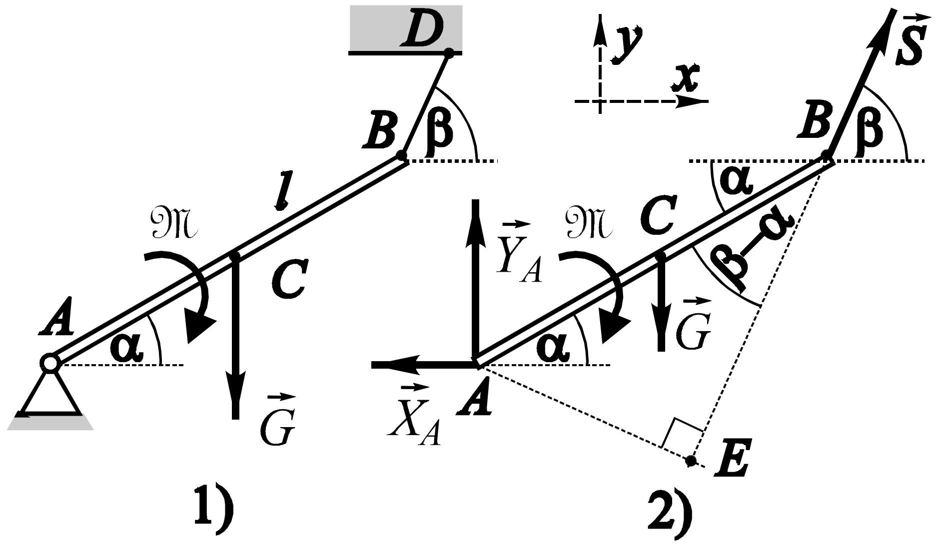
$$\sum X_i = F_A - S\cos\alpha = 0 \quad \Rightarrow$$

$$S = \frac{P\cos\beta}{2\sin(\beta-\alpha)}$$

$$\sum Y_i = F_B - P - S\sin\alpha = 0 \quad \Rightarrow$$

$$F_B = P + \frac{P\sin\alpha\cos\beta}{2\sin(\beta-\alpha)}$$

Primer 6.4



Homogeni štap AB težine G, dužine l, koji sa horizontalom gradi ugao α, vezan je u tački A zglobno a za njegovu tačku B vezano je uže BD koje sa horizontalom gradi ugao β (Sl.1). Na štap dejstvuje i spreg momenta $\mathfrak{M}$ smera datog na slici. Odrediti sve reakcije veza u zavisnosti od poznatih veličina α, β, G, $\mathfrak{M}$ i l.

$$\sum M_{Ai} = -G \cdot \frac{l}{2}\cos\alpha + S \cdot l \sin(\beta-\alpha) - \mathfrak{M} = 0 \quad \Rightarrow \quad S = \frac{G\cos\alpha}{2\sin(\beta-\alpha)} + \frac{\mathfrak{M}}{l\sin(\beta-\alpha)}$$

$$\sum X_i = -X_A + S\cos\beta = 0 \quad \Rightarrow \quad X_A = \frac{G\cos\alpha\cos\beta}{2\sin(\beta-\alpha)} + \frac{\mathfrak{M}\cos\beta}{l\sin(\beta-\alpha)}$$

$$\sum Y_i = Y_A - G + S\sin\beta = 0 \quad \Rightarrow \quad Y_A = G - \frac{G\cos\alpha\sin\beta}{2\sin(\beta-\alpha)} - \frac{\mathfrak{M}\sin\beta}{l\sin(\beta-\alpha)}$$

Primer 6.5

Poznatih veličina G i l

Odrediti ugao α i reakcije u užadima

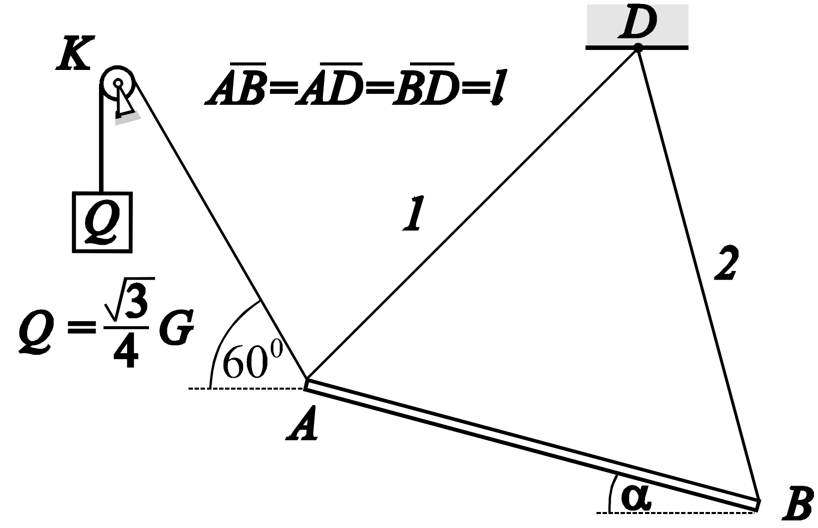
$$\sum M_{D\,i} = G \cdot h_G - Q \cdot h_Q = 0$$

$$h_G = \overline{CD} \sin\alpha = \frac{\sqrt{3}}{2} l \sin\alpha$$

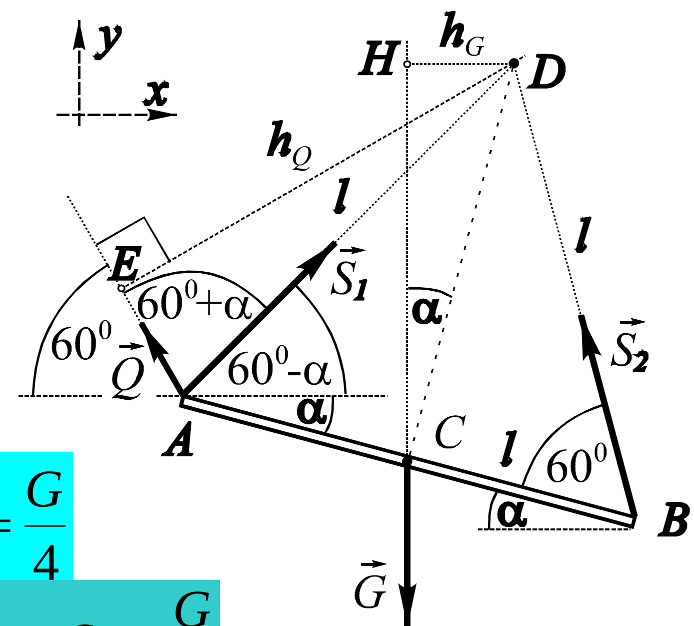
$$h_Q = l \sin\left(60^0 + \alpha\right) = l\left(\frac{\sqrt{3}}{2}\cos\alpha + \frac{1}{2}\sin\alpha\right)$$

$$\Rightarrow \frac{\sqrt{3}}{2} G l \sin\alpha = \frac{\sqrt{3}}{4} G l \left(\frac{\sqrt{3}}{2}\cos\alpha + \frac{1}{2}\sin\alpha\right)$$

$$\sqrt{3}\sin\alpha = \cos\alpha \quad \Rightarrow \tan\alpha = \frac{\sqrt{3}}{3} \Rightarrow \quad \alpha = 30^0$$



1)

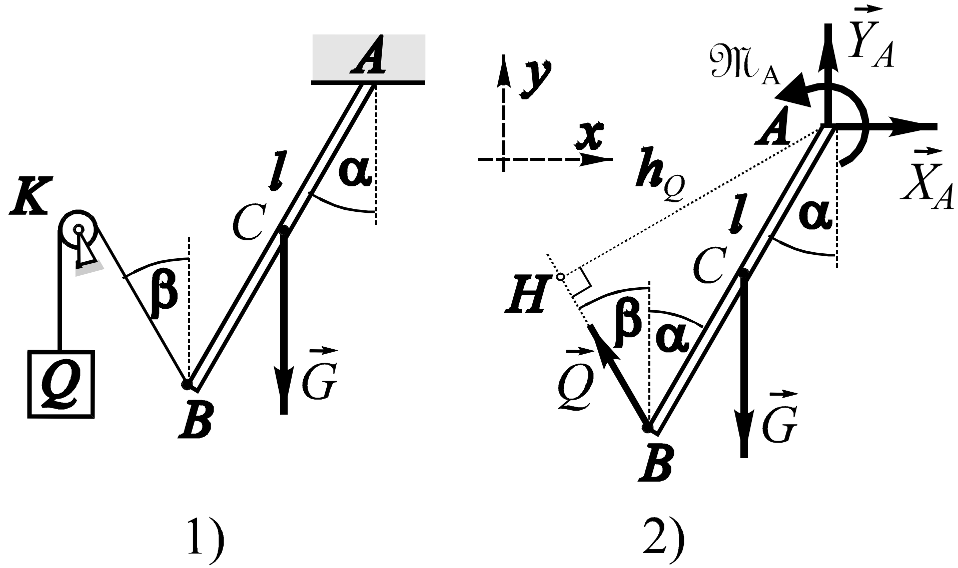


2)

$$\sum X_i = -G\frac{\sqrt{3}}{4}\cos 60^0 + S_1 \cos 30^0 = 0 \Rightarrow S_1 = \frac{G}{4}$$

$$\sum Y_i = G\frac{\sqrt{3}}{4}\sin 60^0 + S_1 \sin 30^0 - G + S_2 = 0 \quad \Rightarrow S_2 = \frac{G}{2}$$

Primer 6.6



Homogeni štap AB težine G, dužine l, koji sa vertikalom gradi ugao α, uklešten je u tački A a za njegovu tačku B je vezano uže koje sa vertikalom gradi ugao β (Sl.1). Uže je prebačeno preko idealnog kotura K a na njegovom drugom kraju je okačen teret težine Q. Odrediti sve reakcije veza u zavisnosti od poznatih veličina α, β, G, Q i l.

Mada to u ovom zadatku ne donosi neku prednost, prvo napišimo momentnu jednačinu za momentnu tačku A

$$\sum M_{Ai} = G\cdot\frac{l}{2}\sin\alpha - Q\cdot l\sin(\alpha+\beta) + \mathfrak{M}_A = 0 \quad \Rightarrow \quad \mathfrak{M}_A = Q\cdot l\sin(\alpha+\beta) - G\cdot\frac{l}{2}\sin\alpha$$

Druga dva uslova ravnoteže odrediće preostale dve nepoznate:

$$\sum X_i = X_A - Q\sin\beta = 0 \quad \Rightarrow \quad X_A = Q\sin\beta$$

$$\sum Y_i = Y_A - G + Q\cos\beta = 0 \quad \Rightarrow \quad Y_A = G - Q\cos\beta$$

Primer 6.8

Poznate veličine: P, G, a, $\mathfrak{M}=Ga/4$

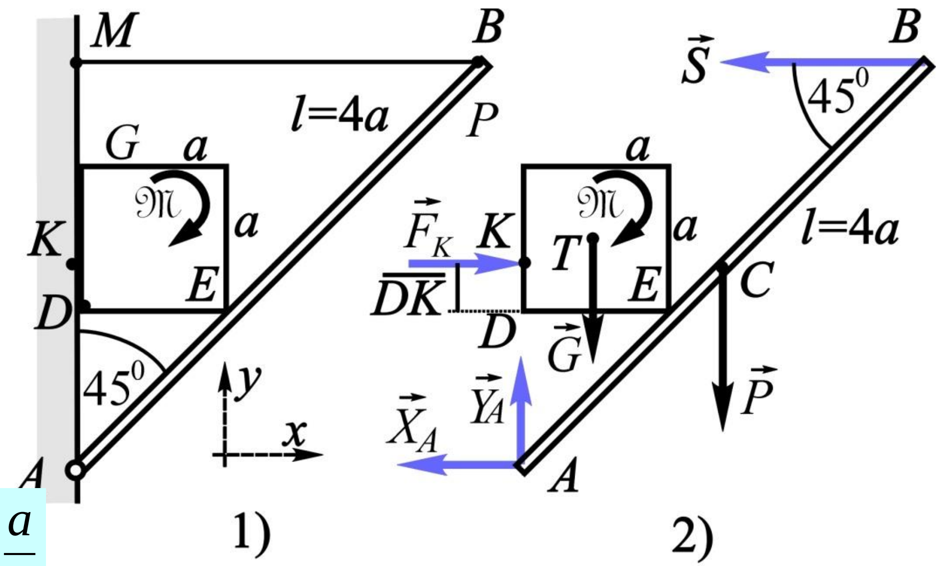
Odrediti sve reakcije veza i $\overline{DK}$

Sl.4

$$\sum Y_i = F_E \sin 45^0 - G = 0$$
$$\Rightarrow F_E = \sqrt{2}G$$

$$\sum X_i = F_K - F_E \cos 45^0 = 0 \Rightarrow F_K = G$$

$$\sum M_{Ei} = G\frac{a}{2} - F_K \overline{DK} - \mathfrak{M} = 0 \;\Rightarrow\; \overline{DK} = \frac{a}{4}$$



Sl.3 $$\sum M_{Ai} = S\cdot 4a\frac{\sqrt{2}}{2} - P\cdot 2a\frac{\sqrt{2}}{2} - F_E\cdot\overline{AE} = 0$$

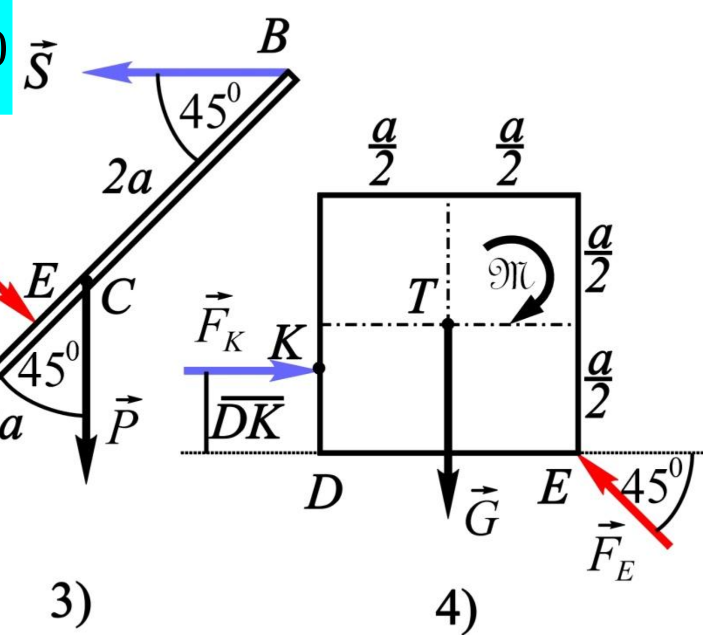
$$\Rightarrow \quad S = \frac{P}{2} + \frac{\sqrt{2}}{2}G$$

$$\overline{AE} = \sqrt{2}a$$

Sl.2

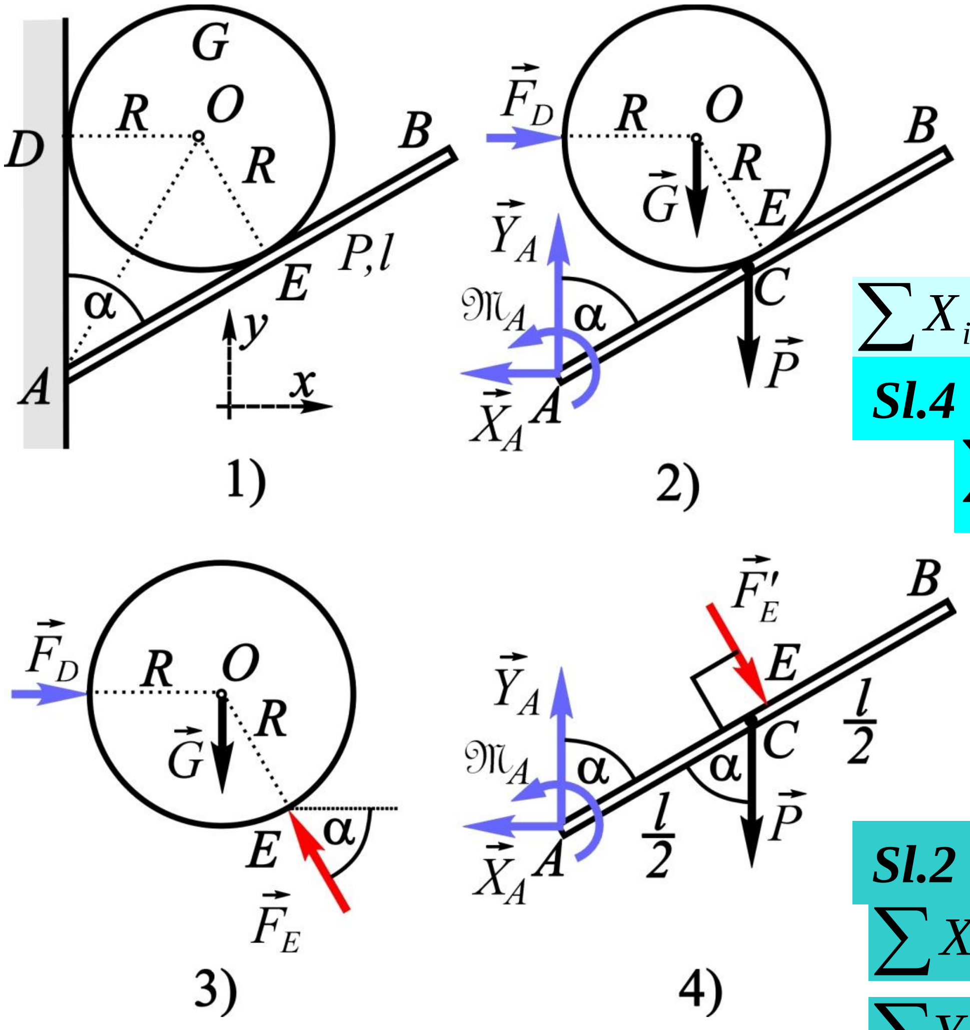
$$\sum X_i = -X_A + F_K - S = 0$$
$$\Rightarrow \quad X_A = G - \frac{\sqrt{2}}{2}G - \frac{P}{2}$$

$$\sum Y_i = Y_A - P - G = 0 \Rightarrow Y_A = P + G$$



Primer 6.9

Poznate veličine: P, G, R i α
Odrediti sve reakcije veza?



Sl.3

$$\sum Y_i = F_E \sin\alpha - G = 0$$

$$\Rightarrow F_E = \frac{G}{\sin\alpha}$$

$$\sum X_i = F_D - F_E \cos\alpha = 0 \Rightarrow F_D = G \cot\alpha$$

Sl.4

$$\sum M_{Ai} = -P \cdot \frac{l}{2} \sin\alpha - F_E \cdot \overline{AE} + \mathfrak{M}_A = 0$$

$$\overline{AE} = R \cot\frac{\alpha}{2}$$

$$\Rightarrow \mathfrak{M}_A = \frac{Pl}{2} \sin\alpha + \frac{GR}{\sin\alpha} \cot\frac{\alpha}{2}$$

Sl.2

$$\sum X_i = -X_A + F_D = 0 \Rightarrow X_A = G \cot\alpha$$

$$\sum Y_i = Y_A - P - G = 0 \Rightarrow Y_A = P + G$$

Primer 6.10

Poznate veličine: P, G, F, a i b

Odrediti uglove α i β i reakcije veza?

Sl.4

$$\sum M_{Ai} = -G \cdot b\sin\beta + F \cdot 2b\cos\beta = 0$$

$$\ldots\ldots\ldots \Big| \cdot \frac{1}{b\cos\beta} \qquad \Rightarrow \quad \tan\beta = \frac{2F}{G}$$

$$\sum Y_i = Y_A - G = 0 \quad \Rightarrow \quad Y_A = G$$

$$\sum X_i = -X_A + F = 0 \quad \Rightarrow \quad X_A = F$$

Sl.3

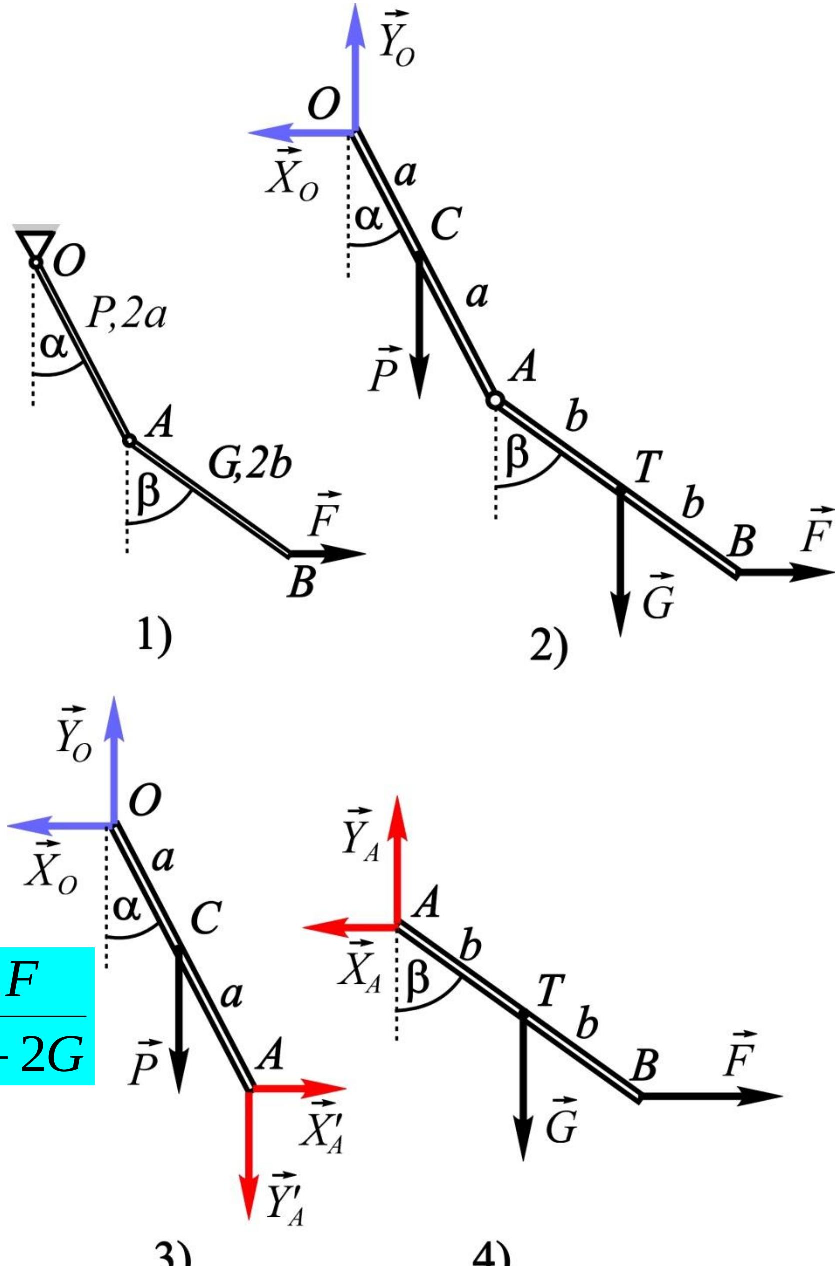
$$\sum M_{Oi} = -P \cdot a\sin\alpha - Y_A \cdot 2a\sin\alpha +$$

$$+ X_A \cdot 2a\cos\alpha = 0 \Big| \cdot \frac{1}{a\cos\alpha} \Rightarrow \tan\alpha = \frac{2F}{P+2G}$$

Sl.2

$$\sum X_i = -X_O + F = 0 \quad \Rightarrow \quad X_O = F$$

$$\sum Y_i = Y_O - P - G = 0 \quad \Rightarrow \quad Y_O = P + G$$



Primer 6.11

Poznate veličine: P, G, F i a

Odrediti reakcije veza u A i B ?

Sl.2 $\sum M_{Ai} = 0 \Rightarrow$

$$-G \cdot \frac{a}{2} - P \cdot \frac{3}{2}a + F \cdot \sqrt{3}a + Y_B \cdot 2a = 0$$

$$\Rightarrow \quad Y_B = \frac{G}{4} + \frac{3}{4}P - \frac{\sqrt{3}}{2}F$$

$$\sum Y_i = Y_A + Y_B - G - P = 0$$

$$\Rightarrow \quad Y_A = \frac{3}{4}G + \frac{P}{4} + \frac{\sqrt{3}}{2}F$$

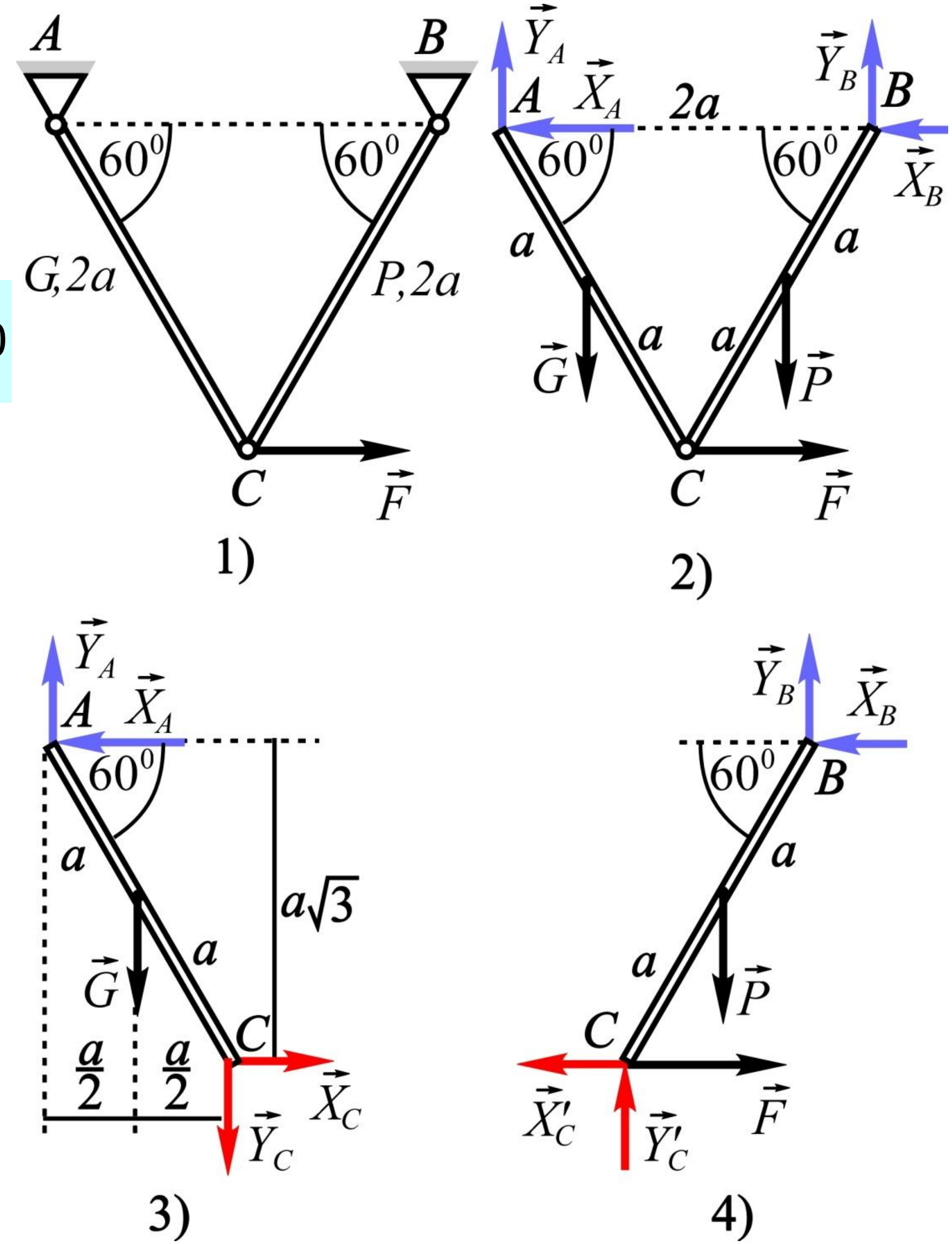
Sl.3 $\sum M_{Ci} = 0 \Rightarrow$

$$G \cdot \frac{a}{2} - Y_A \cdot a + X_A \cdot \sqrt{3}a = 0$$

$$\Rightarrow \quad X_A = \frac{\sqrt{3}(G+P) + 6F}{12}$$

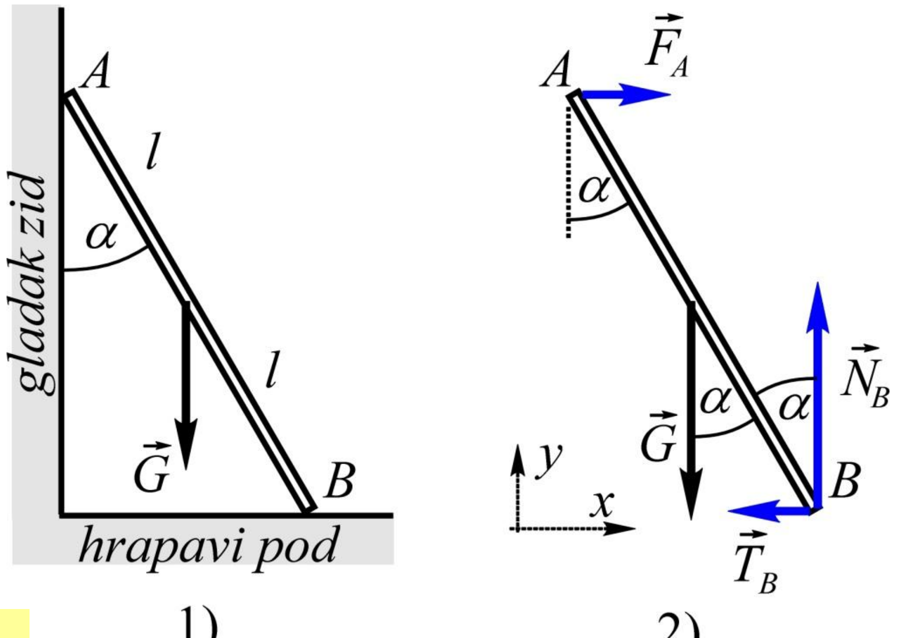
Sl.2 $\sum X_i = -X_A - X_B + F = 0$

$$\Rightarrow \quad X_B = \frac{-\sqrt{3}(G+P) + 6F}{12}$$



Primer 9.1 Poznate veličine: G, l, α

Odrediti sve reakcije veza?



$$\sum M_{Bi} = G \cdot l \sin\alpha - F_A \cdot 2l\cos\alpha = 0$$

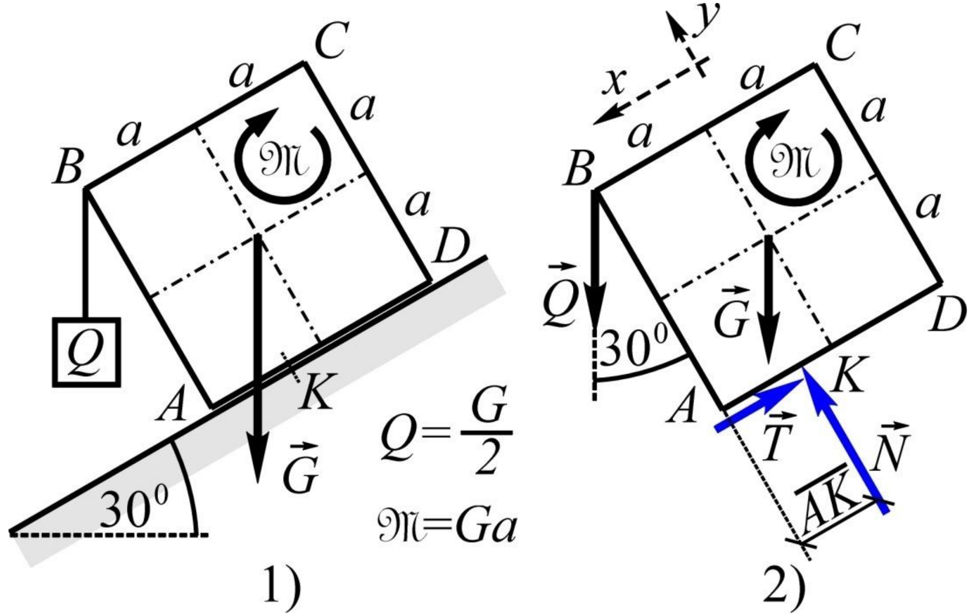
$$\Rightarrow \quad F_A = \frac{G}{2}\tan\alpha$$

$$\sum X_i = F_A - T_B = 0 \quad \Rightarrow \quad T_B = \frac{G}{2}\tan\alpha$$

$$\sum Y_i = -G + N_B = 0 \quad \Rightarrow \quad N_B = G$$

Primer 9.2

Homogena kvadratna ploča *ABCD* težine *G*, stranice *2a*, nalazi se na hrapavoj strmoj ravni koja sa horizontalom gradi ugao od 30^0. Za teme *B* ploče, posredstvom užeta, okačen je teret težine $Q=G/2$. Na ploču dejstvuje i spreg momenta $\mathrm{M} = Ga$, smera datog na slici. Za prikazan ravnotežni položaj, u zavisnosti od poznatih veličina *G* i *a*, odrediti komponente reakcije podloge, koja dejstvuje u tački *K*, kao i mesto te reakcije (rastojanje $\overline{AK}$).



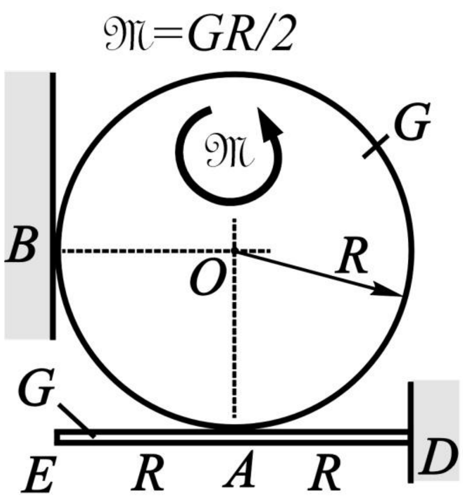
Sl.2

$$\sum X_i = -T + G\sin 30^0 + Q\sin 30^0 = 0 \quad \Rightarrow \quad T = \frac{3}{4}G$$

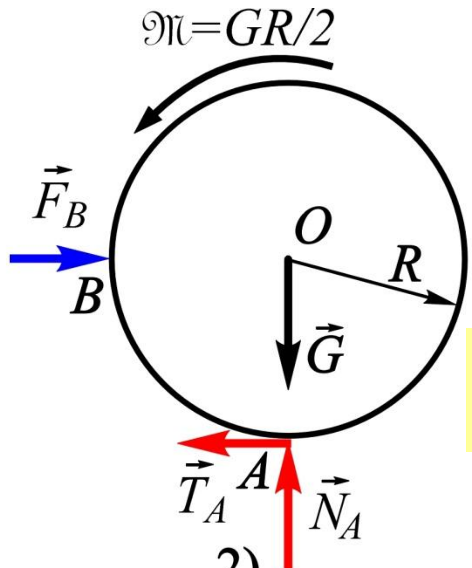
$$\sum Y_i = N - G\cos 30^0 - Q\cos 30^0 = 0 \quad \Rightarrow \quad N = \frac{3\sqrt{3}}{4}G$$

$$\sum M_{Ai} = -G\cdot\left(a\cos 30^0 - a\sin 30^0\right) + N\cdot\overline{AK} + Q\cdot 2a\sin 30^0 - \mathrm{M} = 0 \Rightarrow \overline{AK} = \frac{2}{3}a$$

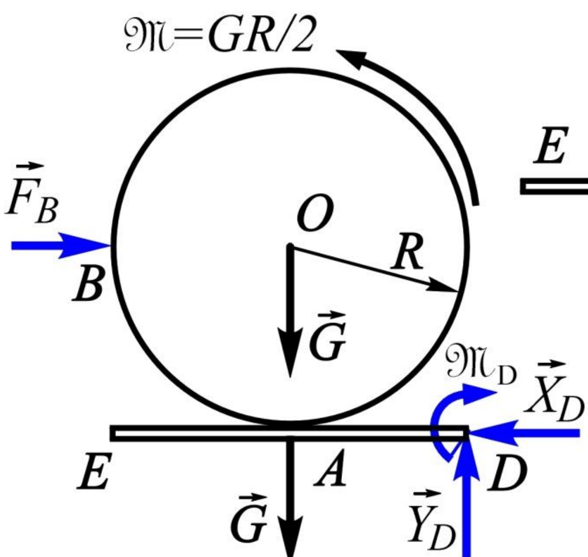
Primer 9.3



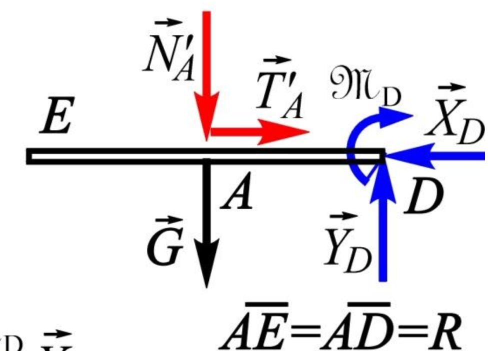
1)



2)



3)



4)

Vertikalan zid je gladak.

Veza u tački *A* je neidealna (hrapavi su i disk i štap).

Poznate veličine su: *G*, *R*

Odrediti sve reakcije veza?

Sl.2

$$\sum M_{Ai} = -F_B \cdot R + \mathfrak{M} = 0 \Rightarrow F_B = \frac{G}{2}$$

$$\sum X_i = F_B - T_A = 0 \quad \Rightarrow \quad T_A = \frac{G}{2}$$

$$\sum Y_i = N_A - G = 0 \quad \Rightarrow \quad N_A = G$$

Sl.4

$$\sum M_{Di} = N_A \cdot R + G \cdot R - \mathfrak{M}_D = 0$$

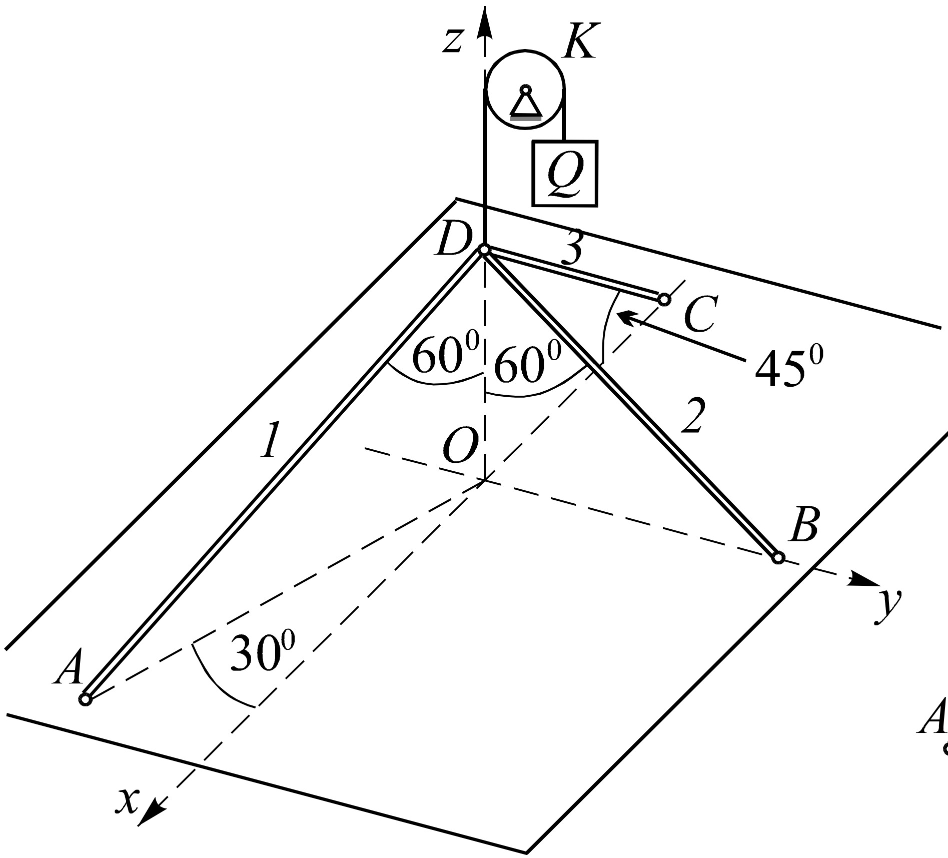
$$\Rightarrow \quad \mathfrak{M}_D = 2GR$$

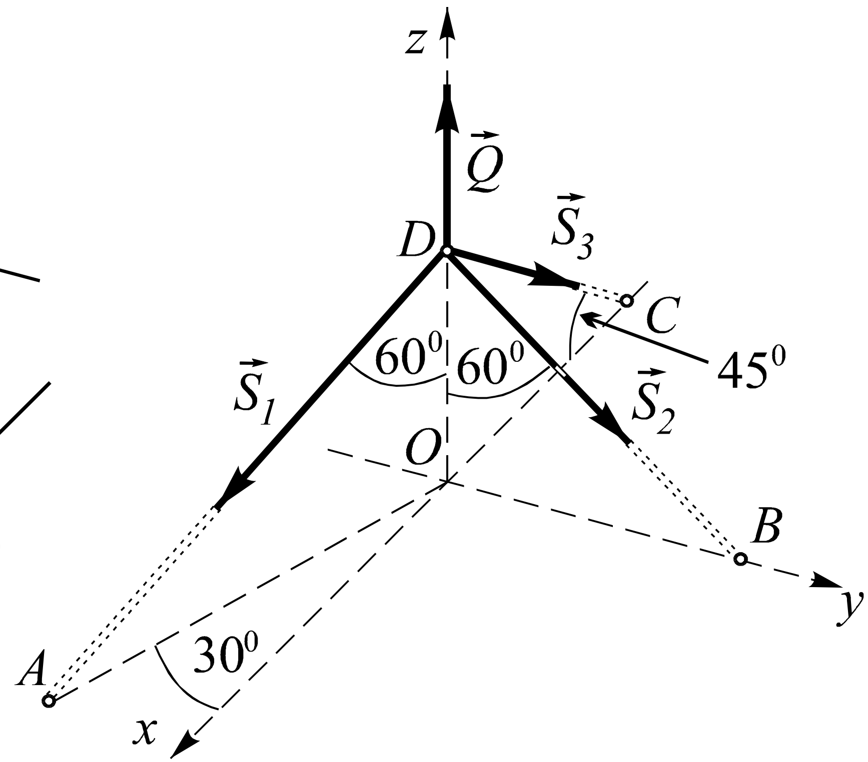
Sl.3

$$\sum X_i = -X_D + F_B = 0 \Rightarrow X_D = \frac{G}{2}$$

$$\sum Y_i = Y_D - G - G = 0 \Rightarrow Y_D = 2G$$

Primer 4.9





$$X_2 = 0, \quad Y_2 = S_2 \sin 60^0, \quad Z_2 = -S_2 \cos 60^0$$

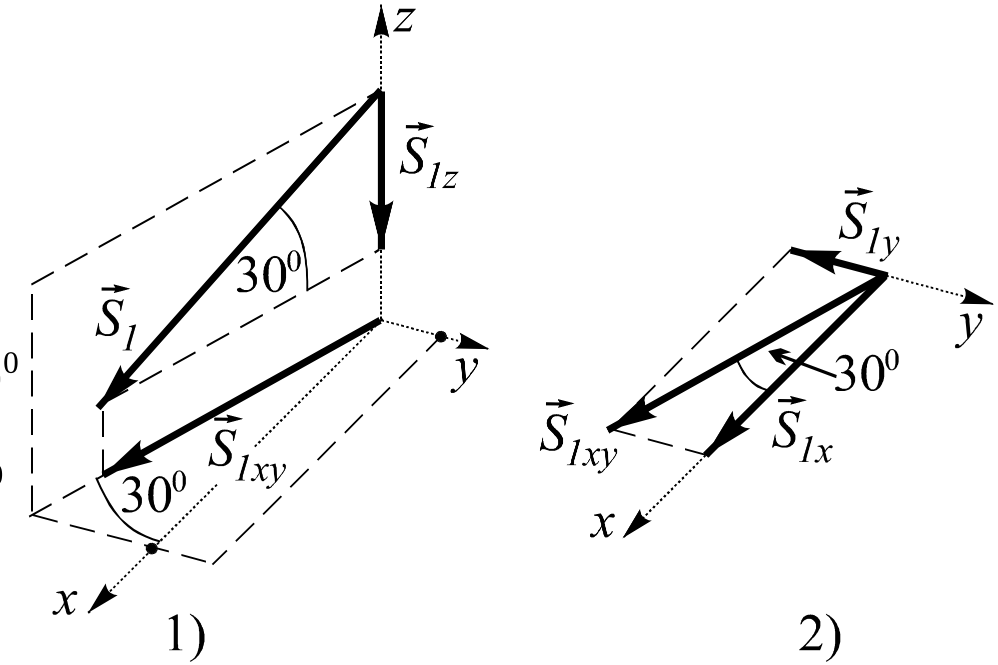
$$X_3 = -S_3 \cos 45^0, \quad Y_3 = 0, \quad Z_3 = -S_3 \sin 45^0$$

$$\vec{S}_i = X_i\vec{i} + Y_i\vec{j} + Z_i\vec{k}$$

$$S_{1z} = S_1 \sin 30^0, \quad S_{1xy} = S_1 \cos 30^0$$

$$S_{1x} = S_{1xy} \cos 30^0 = S_1 \cos 30^0 \cos 30^0$$

$$S_{1y} = S_{1xy} \sin 30^0 = S_1 \cos 30^0 \sin 30^0$$



Razlaganje sile $\vec{S}_1$ *na komponente*

$$\sum X_i = \frac{3}{4}\boldsymbol{S_1} - \frac{\sqrt{2}}{2}\boldsymbol{S_3} = 0$$

$$\sum Y_i = -\frac{\sqrt{3}}{4}\boldsymbol{S_1} + \frac{\sqrt{3}}{2}\boldsymbol{S_2} = 0$$

$$\Rightarrow \quad \boldsymbol{S_3} = \frac{3}{2\sqrt{2}}\boldsymbol{S_1}, \quad \boldsymbol{S_2} = \frac{1}{2}\boldsymbol{S_1}$$

$$\sum Z_i = -\frac{1}{2}\boldsymbol{S_1} - \frac{1}{2}\boldsymbol{S_2} - \frac{\sqrt{2}}{2}\boldsymbol{S_3} + Q = 0$$

$$\boldsymbol{S_1} = \frac{2}{3}Q, \quad \boldsymbol{S_2} = \frac{1}{3}Q, \quad \boldsymbol{S_3} = \frac{\sqrt{2}}{2}Q$$

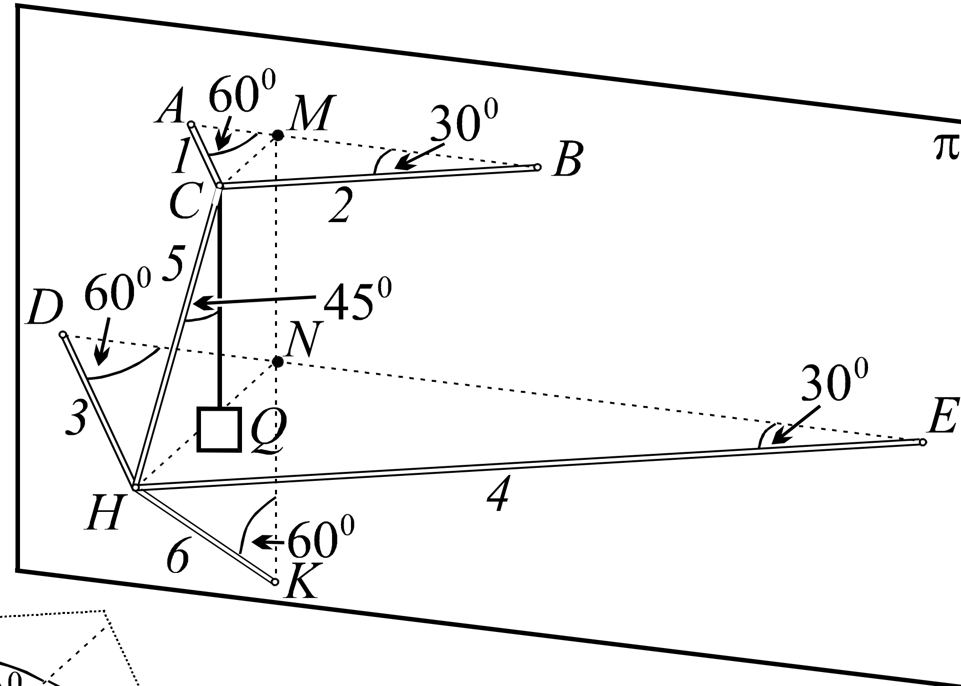
Primer 4.10

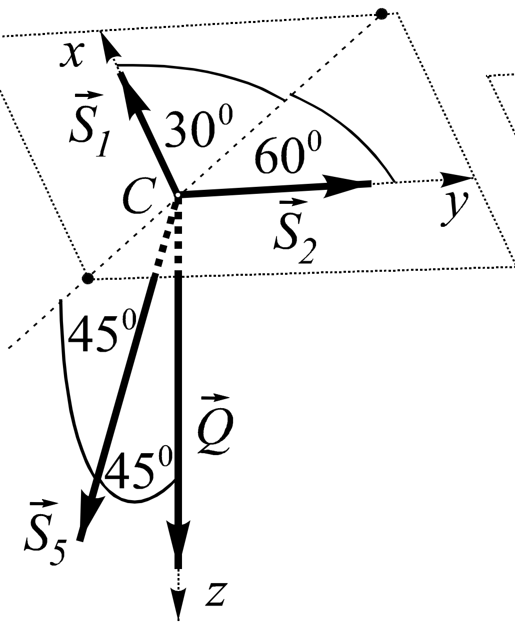
Određivanje projekcija sile $\vec{S}_5$

$$X_5 = -S_5 \sin 45^0 \cos 30^0$$

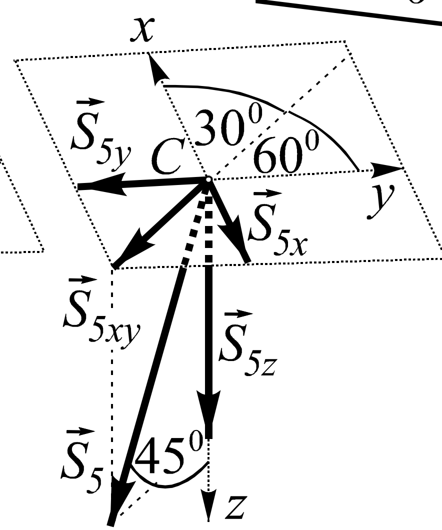
$$Y_5 = -S_5 \sin 45^0 \cos 60^0$$

$$Z_5 = S_5 \cos 45^0$$





1)



2)

Uslovi ravnoteže tačke C

$$\sum X_i = \boldsymbol{S_1} - \frac{\sqrt{2}\sqrt{3}}{4}\boldsymbol{S_5} = 0$$

$$\sum Y_i = \boldsymbol{S_2} - \frac{\sqrt{2}}{4}\boldsymbol{S_5} = 0$$

$$\sum Z_i = \frac{\sqrt{2}}{2}\boldsymbol{S_5} + Q = 0$$

$$\Rightarrow \quad \boldsymbol{S_5} = -\sqrt{2}Q,\ \ \boldsymbol{S_1} = -\frac{\sqrt{3}}{2}Q,\ \ \boldsymbol{S_2} = -\frac{1}{2}Q$$

zbog $\vec{S}_5' = -\vec{S}_5 \ \Rightarrow$

$$X_5' = -X_5 = \frac{\sqrt{2}\sqrt{3}}{4}S_5 = -\frac{\sqrt{3}}{2}Q$$

$$Y_5' = -Y_5 = \frac{\sqrt{2}}{4}S_5 = -\frac{1}{2}Q$$

$$Z_5' = -Z_5 = -\frac{\sqrt{2}}{2}S_5 = Q$$

Uslovi ravnoteže tačke H

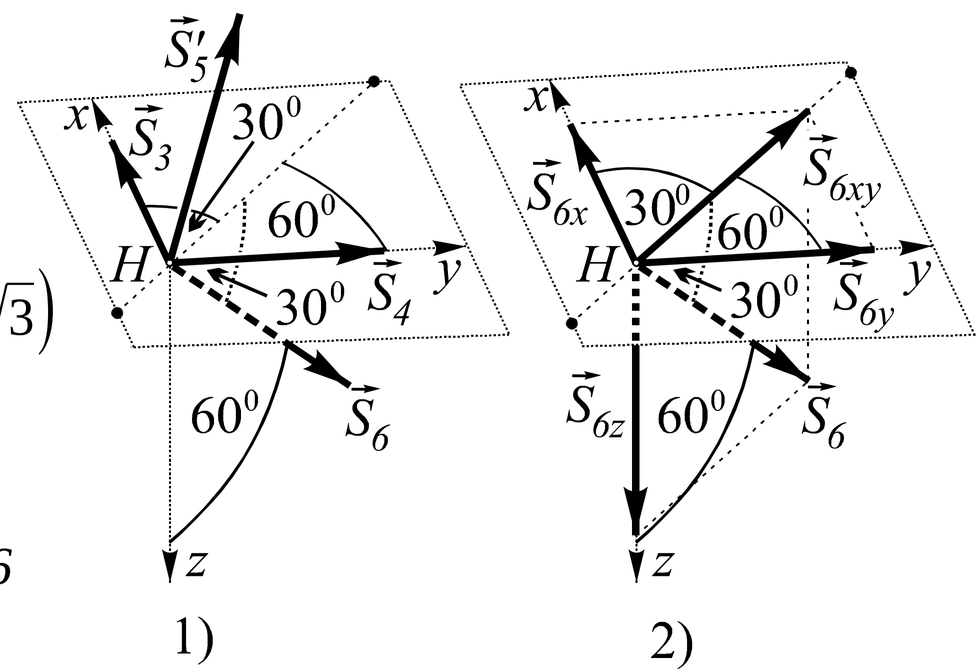
$$\sum X_i = \boldsymbol{S_3} + \frac{3}{4}\boldsymbol{S_6} - \frac{\sqrt{3}}{2}Q = 0$$

$$\sum Y_i = \boldsymbol{S_4} + \frac{\sqrt{3}}{4}\boldsymbol{S_6} - \frac{1}{2}Q = 0$$

$$\sum Z_i = \frac{1}{2}\boldsymbol{S_6} + Q = 0$$

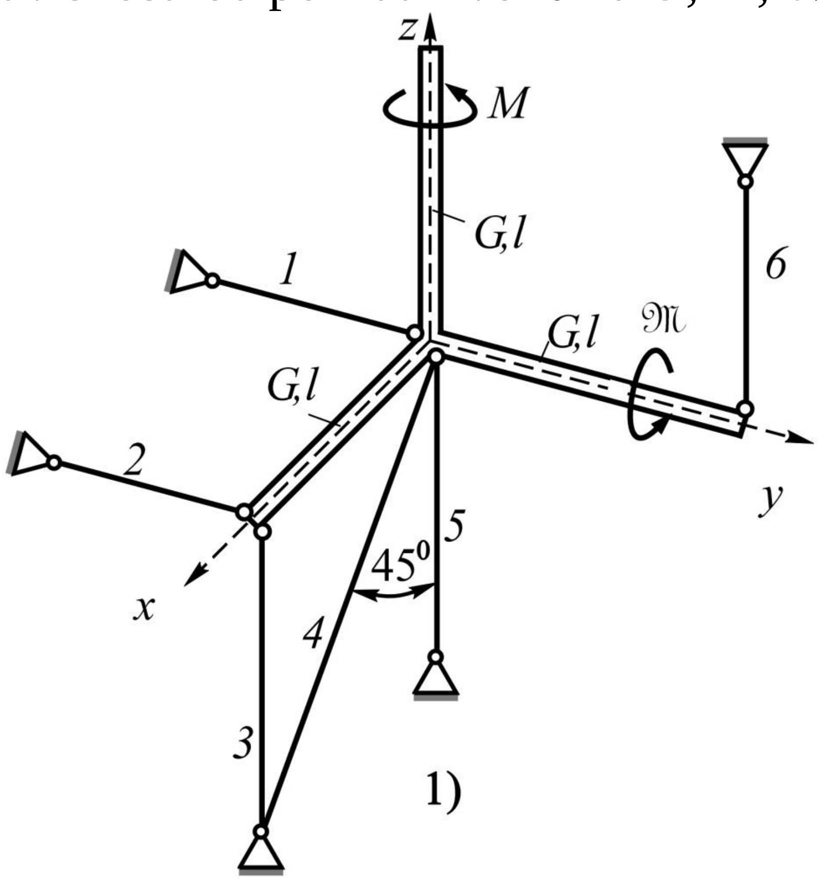
$\Rightarrow$

$$\boldsymbol{S_6} = -2Q,\ \boldsymbol{S_3} = \frac{Q}{2}\left(3+\sqrt{3}\right),\ \boldsymbol{S_4} = \frac{Q}{2}\left(1+\sqrt{3}\right)$$

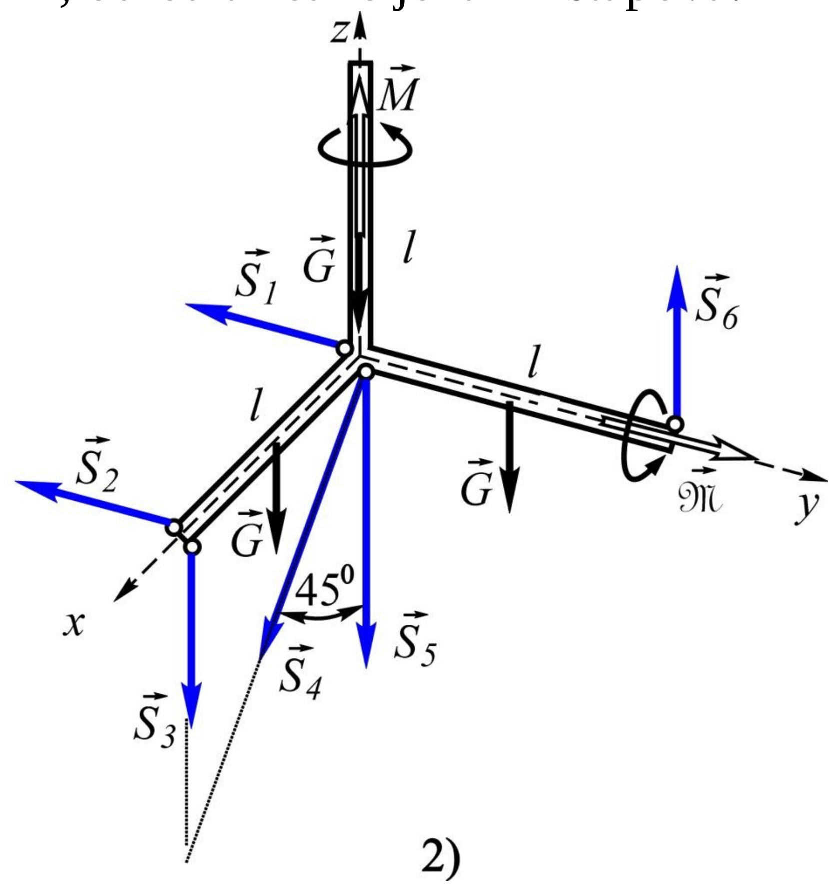


Predznaci u dobijenim rešenjima ukazuju na to da su štapovi *1*, *2*, *5* i *6* pritisnuti, a štapovi *3* i *4* zategnuti.

Primer 10.3 Kruto telo sačinjeno od tri kruto spojena međusobno upravna istovetna homogena štapa, težina *G*, dužina *l*, održava u ravnoteži šest lakih štapova kao što je na slici 1 prikazano. Na teški vertikalni štap dejstvuje spreg momenta *M*, koji leži u ravni upravnoj na taj štap, smera datog na slici. Na teški štap, koji ima pravac ose *y*, dejstvuje spreg momenta $\mathfrak{M}$ koji leži u ravni upravnoj na taj štap, smera datog na slici. Za prikazan ravnotežni položaj, u zavisnosti od poznatih veličina *G*, *M*, $\mathfrak{M}$ i *l*, odrediti reakcije lakih štapova.



1)



2)

$$\sum X_i = S_4 \frac{\sqrt{2}}{2} = 0 \Rightarrow \quad S_4 = 0$$

$$\sum Y_i = -S_1 - S_2 = 0 \Rightarrow \quad S_1 = -\frac{M}{l}$$

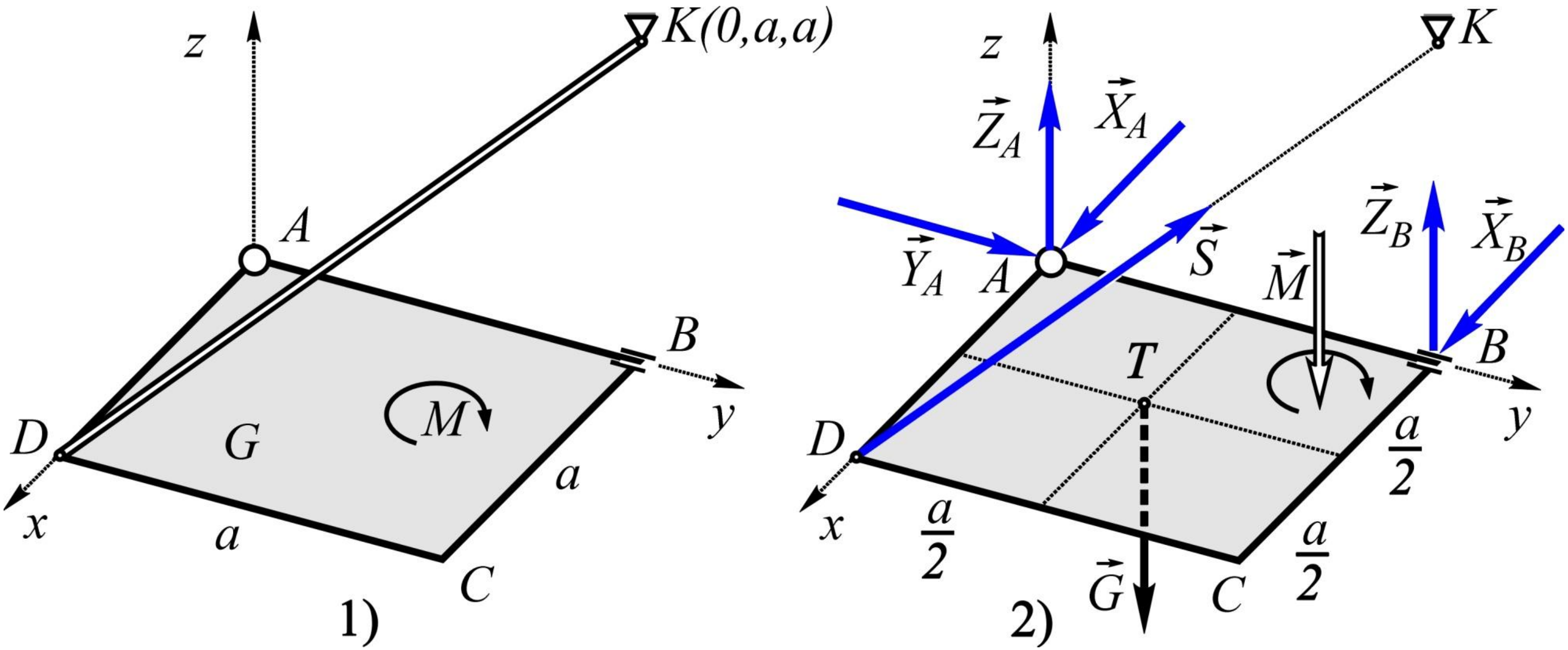
$$\sum Z_i = -S_3 - S_4 \frac{\sqrt{2}}{2} - S_5 + S_6 - 3G = 0 \quad \Rightarrow S_5 = \frac{\mathfrak{M}}{l} - 2G$$

$$\sum M_{x\,i} = -G \cdot \frac{l}{2} + S_6 \cdot l = 0 \quad \Rightarrow \quad S_6 = \frac{G}{2}$$

$$\sum M_{y\,i} = G \cdot \frac{l}{2} + S_3 \cdot l + \mathfrak{M} = 0 \quad \Rightarrow \quad S_3 = -\frac{\mathfrak{M}}{l} - \frac{G}{2}$$

$$\sum M_{z\,i} = -S_2 \cdot l + M = 0 \quad \Rightarrow \quad S_2 = \frac{M}{l}$$

Primer 10.4 Horizontalna homogena kvadratna ploča *ABCD*, težine *G*, stranice *a*, vezana je u tački *A* sfernim zglobom a u tački *B* cilindričnim (Sl.1). Za tačku *D* ploče vezan je laki štap *KD*, gde su zadate koordinate *K(0,a,a)*. Na ploču dejstvuje spreg momenta *M* koji leži u ravni ploče smera datog na slici. Za prikazan ravnotežni položaj, u zavisnosti od poznatih veličina *G*, *M* i *a*, odrediti reakcije veza u tačkama *A* i *B* kao i silu u lakom štapu *KD*.



$$D(a,0,0),\quad K(0,a,a)\quad \Rightarrow \overrightarrow{DK} = -a\vec{i} + a\vec{j} + a\vec{k}$$

$$\overline{DK} = \sqrt{(-a)^2 + a^2 + a^2} = a\sqrt{3}$$

$$x_D = a,$$
$$y_D = z_D = 0$$

$$\vec{S} = \frac{S}{\overline{DK}}\overrightarrow{DK} = \frac{S}{a\sqrt{3}}\left(-a\vec{i} + a\vec{j} + a\vec{k}\right)$$

$$\vec{S} = -\frac{S}{\sqrt{3}}\vec{i} + \frac{S}{\sqrt{3}}\vec{j} + \frac{S}{\sqrt{3}}\vec{k}$$

$$X_S = -\frac{S}{\sqrt{3}},\ Y_S = \frac{S}{\sqrt{3}},\ Z_S = \frac{S}{\sqrt{3}}$$

Jednačine ravnoteže:

$$\sum X_i = X_A + X_B - \frac{S}{\sqrt{3}} = 0 \Rightarrow \quad X_A = \frac{M}{a}$$

$$\sum Y_i = Y_A + \frac{S}{\sqrt{3}} = 0 \quad \Rightarrow \quad Y_A = -\frac{G}{2}$$

$$M_x^{\vec{S}} = y_D Z_S - z_D Y_S = 0$$

$$\sum Z_i = Z_A + Z_B - G + \frac{S}{\sqrt{3}} = 0 \quad \Rightarrow Z_A = 0$$

$$M_y^{\vec{S}} = z_D X_S - x_D Z_S = -a\frac{S}{\sqrt{3}}$$

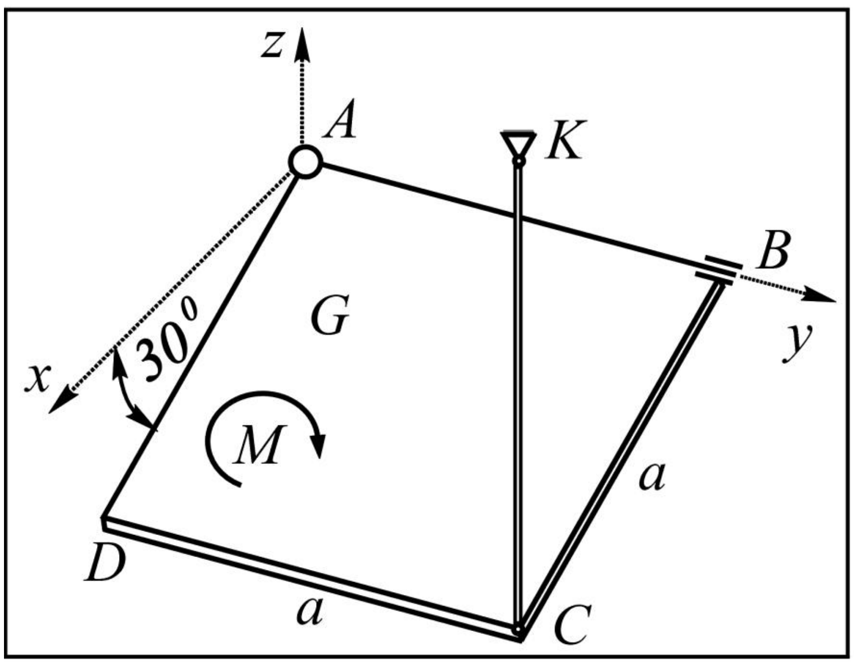
$$\sum M_{xi} = Z_B \cdot a - G \cdot \frac{a}{2} = 0 \quad \Rightarrow \quad Z_B = \frac{G}{2}$$

$$M_z^{\vec{S}} = x_D Y_S - y_D X_S = a\frac{S}{\sqrt{3}}$$

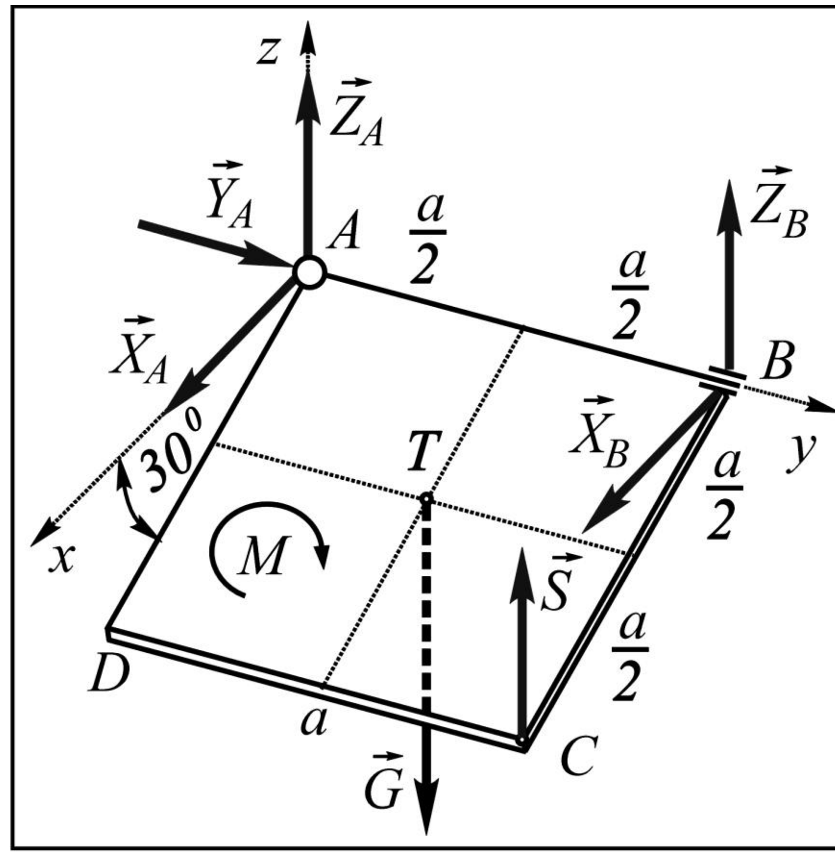
$$\sum M_{yi} = G \cdot \frac{a}{2} - \frac{S}{\sqrt{3}} \cdot a = 0 \quad \Rightarrow \quad S = \frac{\sqrt{3}}{2}G$$

$$\sum M_{zi} = -X_B \cdot a + \frac{S}{\sqrt{3}} \cdot a - M = 0 \quad \Rightarrow \quad X_B = \frac{G}{2} - \frac{M}{a}$$

Primer 10.5 Homogena kvadratna ploča *ABCD* (Sl.1), težine *G*, stranice *a*, koja sa horizontalnom *xAy* ravni gradi ugao od 30^0, vezana je u tački *A* sfernim zglobom a u tački *B* cilindričnim. Za tačku *C* ploče vezan je vertikalni laki štap *KC*. Na ploču dejstvuje spreg momenta M koji leži u ravni ploče smera datog na slici. Za prikazan ravnotežni položaj, u zavisnosti od poznatih veličina *G*, *M* i *a* odrediti reakcije veza u zglobovima *A* i *B* kao i silu u lakom štapu.



1)



2)

Jednačine ravnoteže:

$$\sum X_i = X_A + X_B = 0 \Rightarrow \quad X_A = \frac{\sqrt{3}}{2}\frac{M}{a}$$

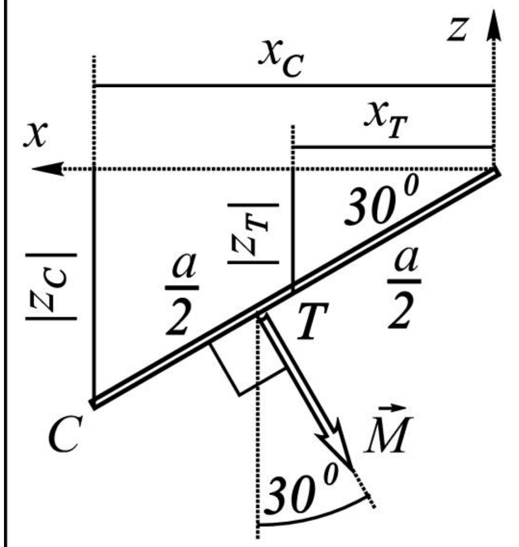
$$\sum Y_i = Y_A = 0 \quad \Rightarrow \quad Y_A = 0$$

$$\sum Z_i = Z_A + Z_B - G + S = 0 \quad \Rightarrow \quad Z_A = \frac{G}{2} - \frac{M}{2a}$$

$$\sum M_{xi} = Z_B \cdot a + S \cdot a - G \cdot \frac{a}{2} - M \cdot \frac{1}{2} = 0 \Rightarrow Z_B = \frac{M}{2a}$$

$$\sum M_{yi} = G \cdot \frac{a}{2}\frac{\sqrt{3}}{2} - S \cdot a\frac{\sqrt{3}}{2} = 0 \quad \Rightarrow \quad S = \frac{G}{2}$$

$$\sum M_{zi} = -X_B \cdot a - M\frac{\sqrt{3}}{2} = 0 \quad \Rightarrow \quad X_B = -\frac{\sqrt{3}}{2}\frac{M}{a}$$



$$x_T = \frac{a}{2}\cos 30^0$$

$$z_T = -\frac{a}{2}\sin 30^0$$

$$x_C = a\cos 30^0$$

$$z_C = -a\sin 30^0$$

$$M_x = -M\sin 30^0$$

$$M_y = 0$$

$$M_z = -M\cos 30^0$$

3)