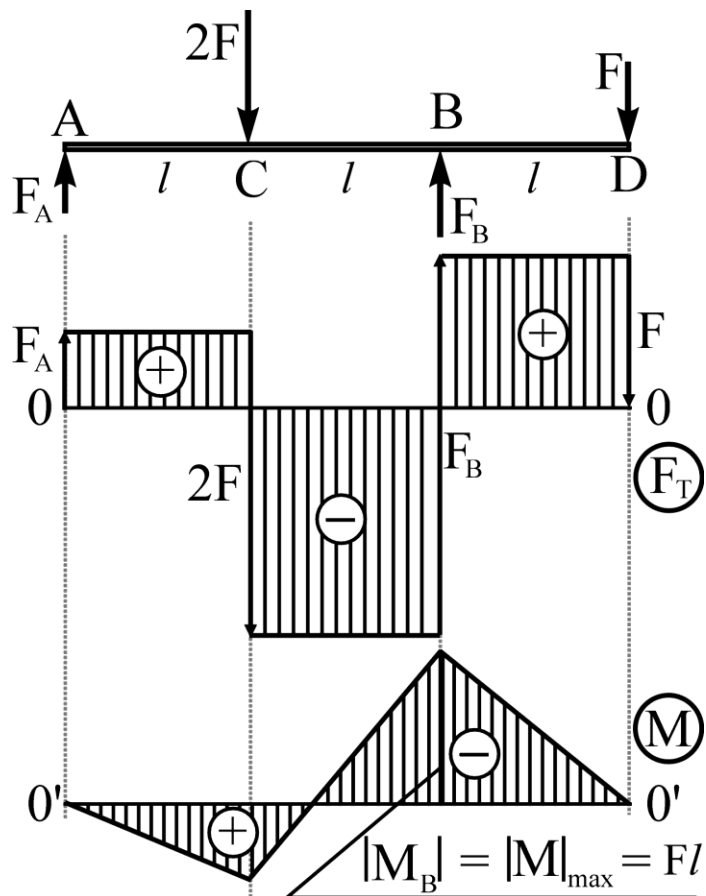
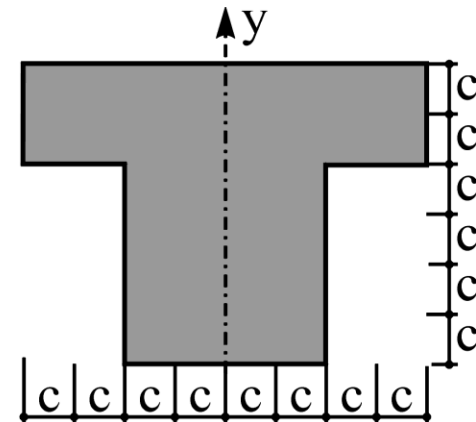
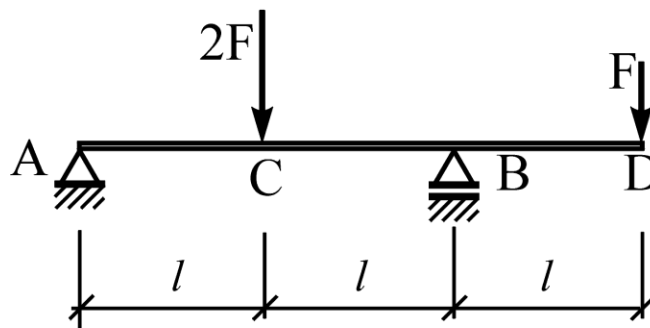


**Primer 3.1** Za zadat gredni nosač: odrediti otpore oslonaca, nacrtati dijagrame momenata savijanja i transverzalnih sila i odrediti maksimalni normalni napon? Poznate veličine su  $F$ ,  $l$  i  $c$ .



**Jednačine ravnoteže:**

$$\sum M_{Ai} = -2F \cdot l + F_B \cdot 2l - F \cdot 3l = 0$$

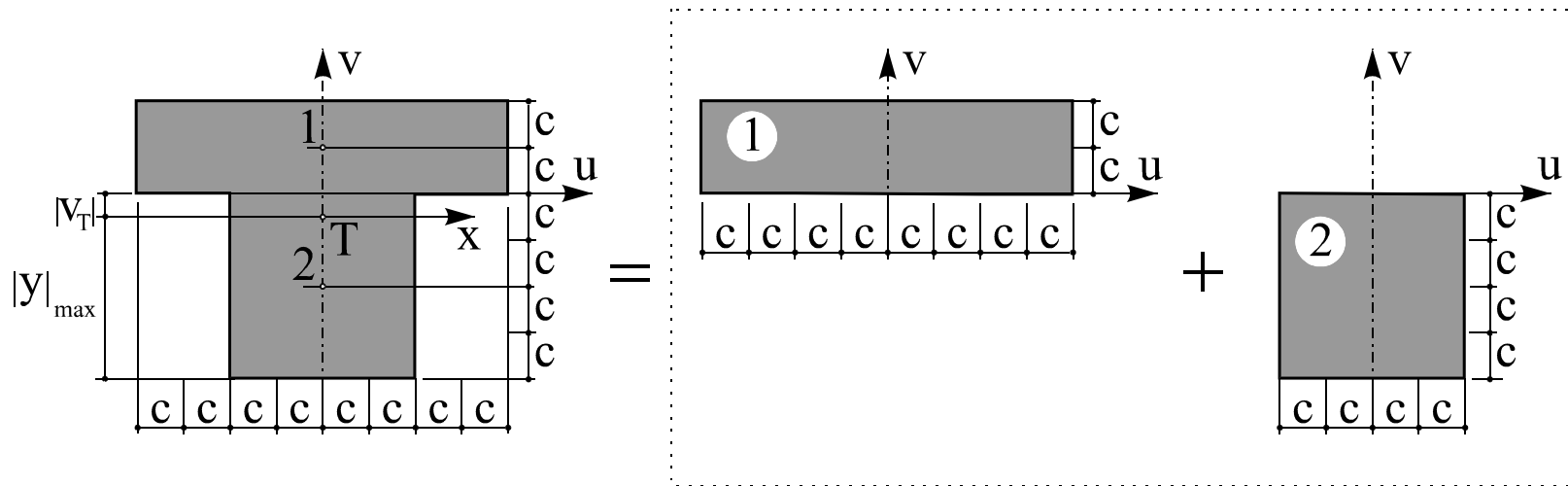
$$\Rightarrow F_B = \frac{5}{2} F,$$

$$\sum Y_i = F_A - 2F + F_B - F = 0 \Rightarrow F_A = \frac{1}{2} F.$$

**Napadni momenti :**

$$M_C = \sum M_{Ci}^l = +F_A \cdot l = +\frac{1}{2} Fl,$$

$$M_B = \sum M_{Bi}^d = -F \cdot l = -Fl.$$



**Određivanje položaja neutralne ose x:**

$$v_T = \frac{\sum A_i \cdot v_i}{A} = \frac{A_1 \cdot v_1 + A_2 \cdot v_2}{A} = \frac{(8c)(2c) \cdot c + (4c)(4c) \cdot (-2c)}{(8c)(2c) + (4c)(4c)} = -\frac{c}{2}, \quad A = 32c^2$$

**Određivanje momenta inercije za neutralnu osu:**

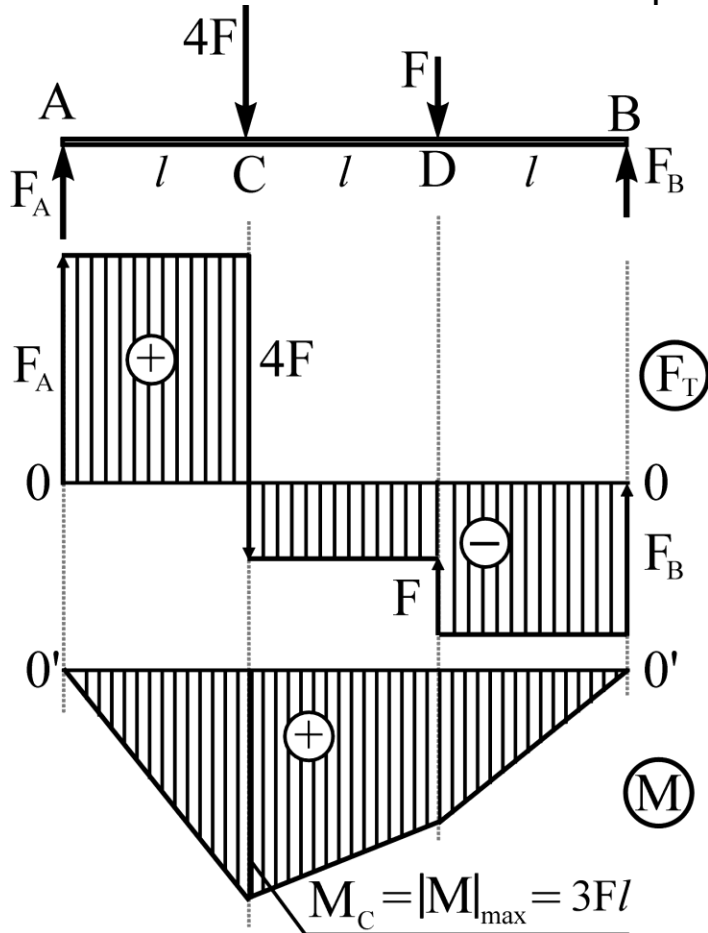
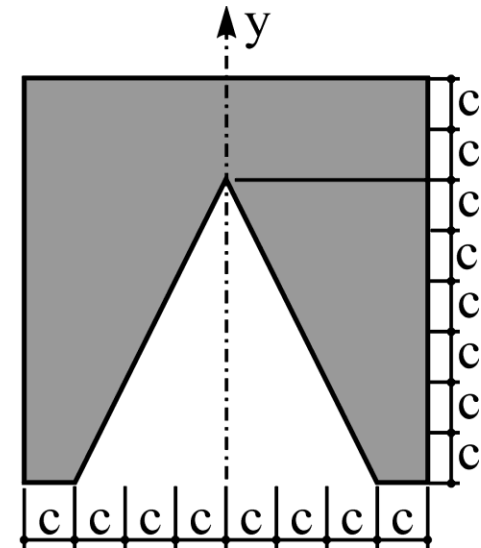
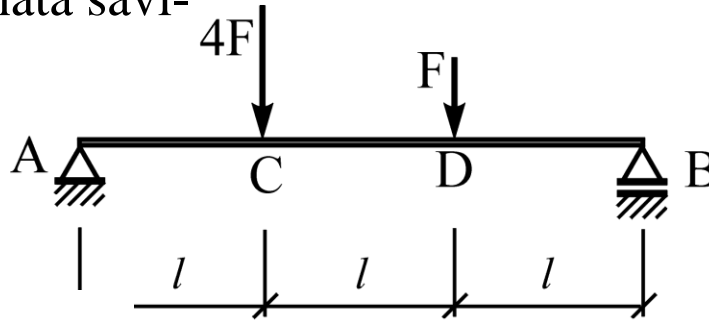
$$I_u = I_u^{(1)} + I_u^{(2)} = \frac{(8c) \cdot (2c)^3}{3} + \frac{(4c) \cdot (4c)^3}{3} = \frac{320}{3} c^4,$$

$$I_x = I_u - A \cdot |v_T|^2 = \frac{320}{3} c^4 - 32c^2 \cdot \frac{c^2}{4} = \frac{296}{3} c^4.$$

**Određivanje otpornog momenta i maksimalnog normalnog napona:**

$$|y|_{\max} = 4c - |v_T| = 7c/2 \Rightarrow W_x = \frac{I_x}{|y|_{\max}} = \frac{592}{21} c^3 \Rightarrow \sigma_{\max} = \frac{|M|_{\max}}{W_x} = \frac{21}{592} \frac{Fl}{c^3}.$$

**Primer 3.2** Za zadat gredni nosač: odrediti otpore oslonaca, nacrtati dijagrame momenata savijanja i transverzalnih sila i odrediti maksimalni normalni napon? Poznate veličine su  $F$ ,  $l$  i  $c$ .



**Jednačine ravnoteže:**

$$\sum M_{Ai} = -4F \cdot l - F \cdot 2l + F_B \cdot 3l = 0$$

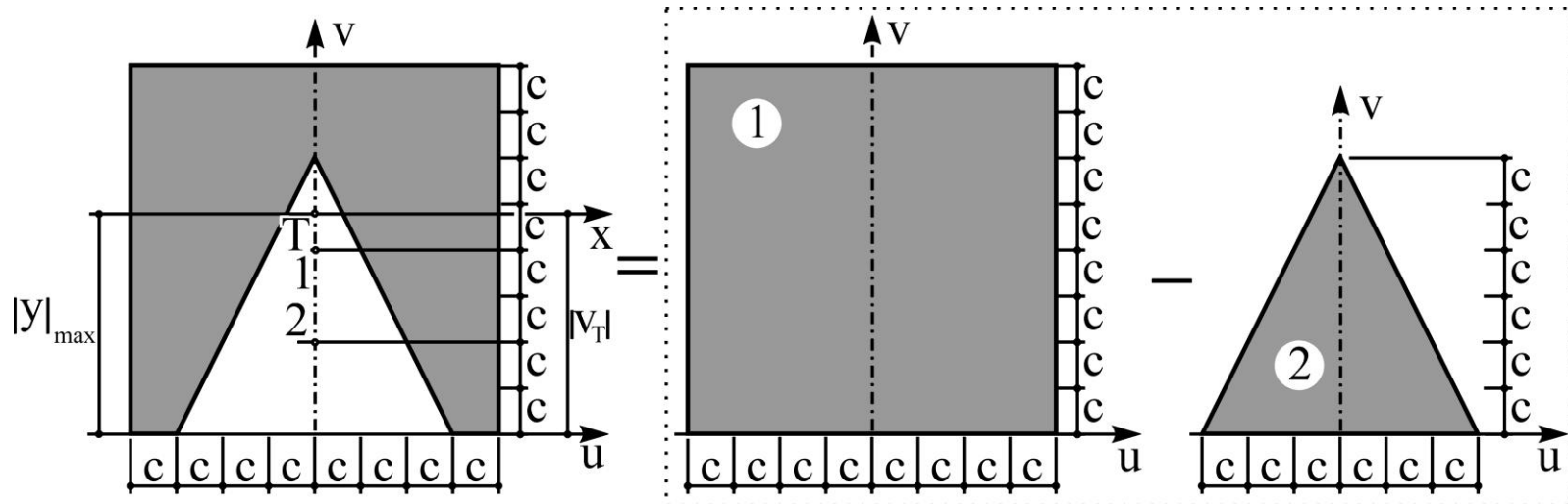
$$\Rightarrow F_B = 2F,$$

$$\sum Y_i = F_A - 4F - F + F_B = 0 \Rightarrow F_A = 3F.$$

**Napadni momenti :**

$$M_C = \sum M_{Ci}^l = +F_A \cdot l = +3Fl,$$

$$M_D = \sum M_{Di}^d = +F_B \cdot l = +2Fl.$$



**Određivanje položaja neutralne ose  $x$ :**

$$v_T = \frac{\sum A_i \cdot v_i}{A} = \frac{A_1 \cdot v_1 - A_2 \cdot v_2}{A} = \frac{(8c)(8c) \cdot 4c - \frac{(6c)(6c)}{2} \cdot 2c}{(8c)(8c) - \frac{(6c)(6c)}{2}} = \frac{110}{23} c, \quad A = 46c^2.$$

**Određivanje momenta inercije za neutralnu osu:**

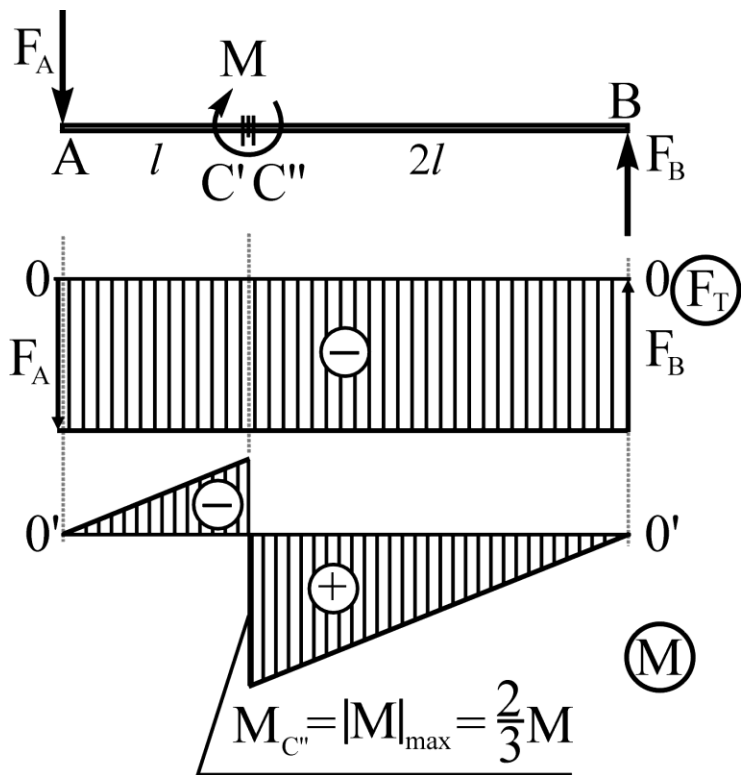
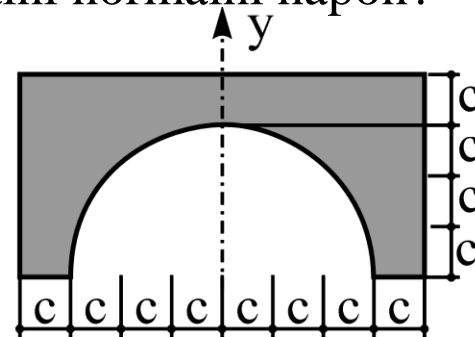
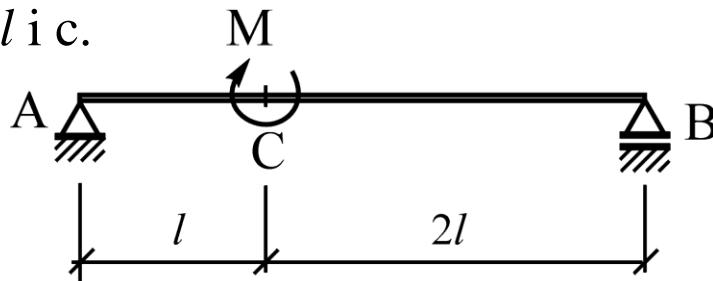
$$I_u = I_u^{(1)} - I_u^{(2)} = \frac{(8c)(8c)^3}{3} - \frac{(6c)(6c)^3}{12} = \frac{3772}{3} c^4,$$

$$I_x = I_u - A \cdot |v_T|^2 = \frac{3772}{3} c^4 - 46c^2 \cdot \frac{110^2}{23^2} c^2 = \frac{14156}{69} c^4.$$

**Određivanje otpornog momenta i maksimalnog normalnog napona:**

$$|y|_{\max} = v_T = 110c/23 \Rightarrow W_x = \frac{I_x}{|y|_{\max}} = \frac{7078}{165} c^3 \Rightarrow \sigma_{\max} = \frac{|M|_{\max}}{W_x} = \frac{495}{7078} \frac{Fl}{c^3}.$$

**Primer 3.3** Za zadan gredni nosač: odrediti otpore oslonaca, nacrtati dijagrame momenata savijanja i transverzalnih sila i odrediti maksimalni normalni napon? Poznate veličine su  $M$ ,  $l$  i  $c$ .



*Jednačine ravnoteže:*

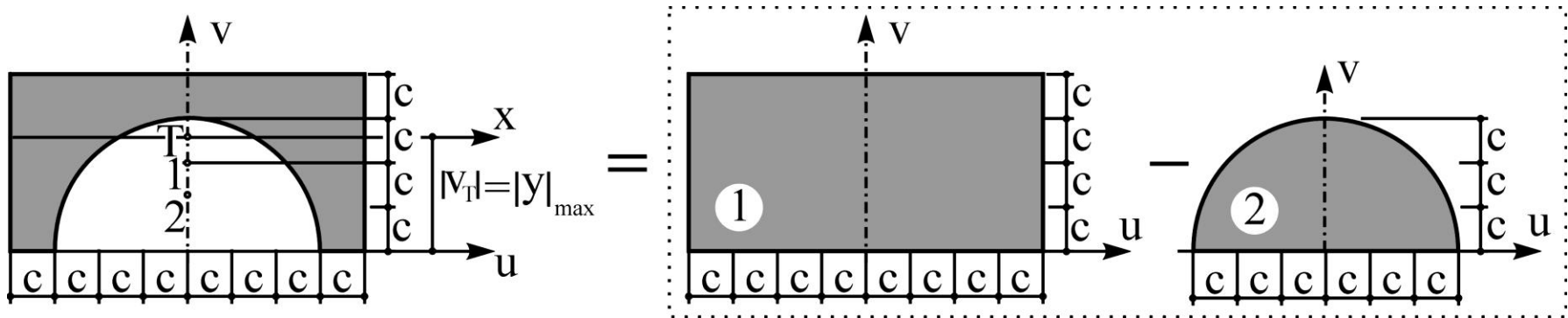
$$\sum M_{Ai} = -M + F_B \cdot 3l = 0 \Rightarrow F_B = M/(3l),$$

$$\sum Y_i = -F_A + F_B = 0 \Rightarrow F_A = M/(3l).$$

*Napadni momenti :*

$$M_{C'} = \sum M_{C'i}^l = -F_A \cdot l = -\frac{M}{3},$$

$$M_{C''} = \sum M_{C''i}^l = -F_A \cdot l + M = +\frac{2}{3}M.$$



**Određivanje položaja neutralne ose x:**

$$v_T = \frac{\sum A_i \cdot v_i}{A} = \frac{A_1 \cdot v_1 - A_2 \cdot v_2}{A} = \frac{(8c)(4c) \cdot 2c - \frac{(3c)^2 \pi}{2} \cdot \frac{4 \cdot 3c}{3\pi}}{(8c)(4c) - \frac{(3c)^2 \pi}{2}} = \frac{92}{64 - 9\pi} c \approx 2,575c,$$

$$A = \left( 32 - \frac{9\pi}{2} \right) c^2 \approx 17,863c^2.$$

**Određivanje momenta inercije za neutralnu osu:**

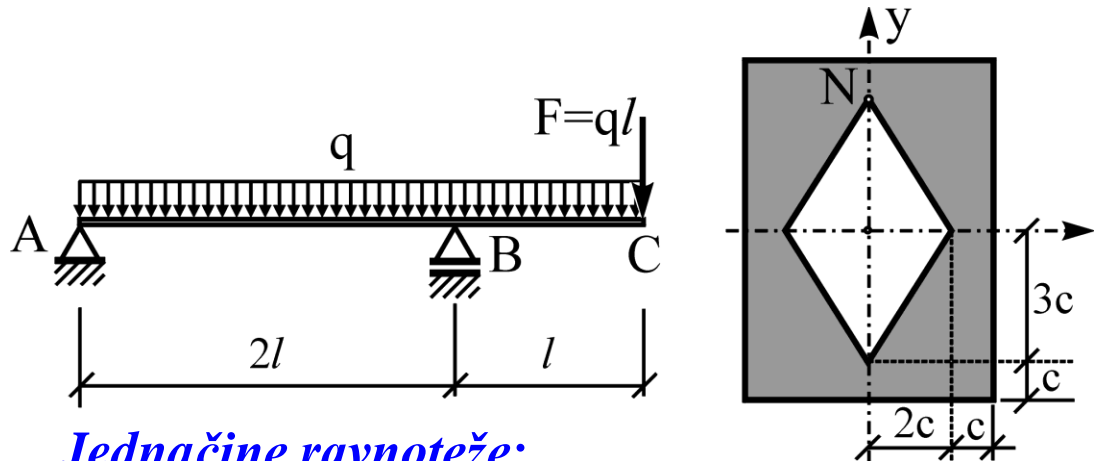
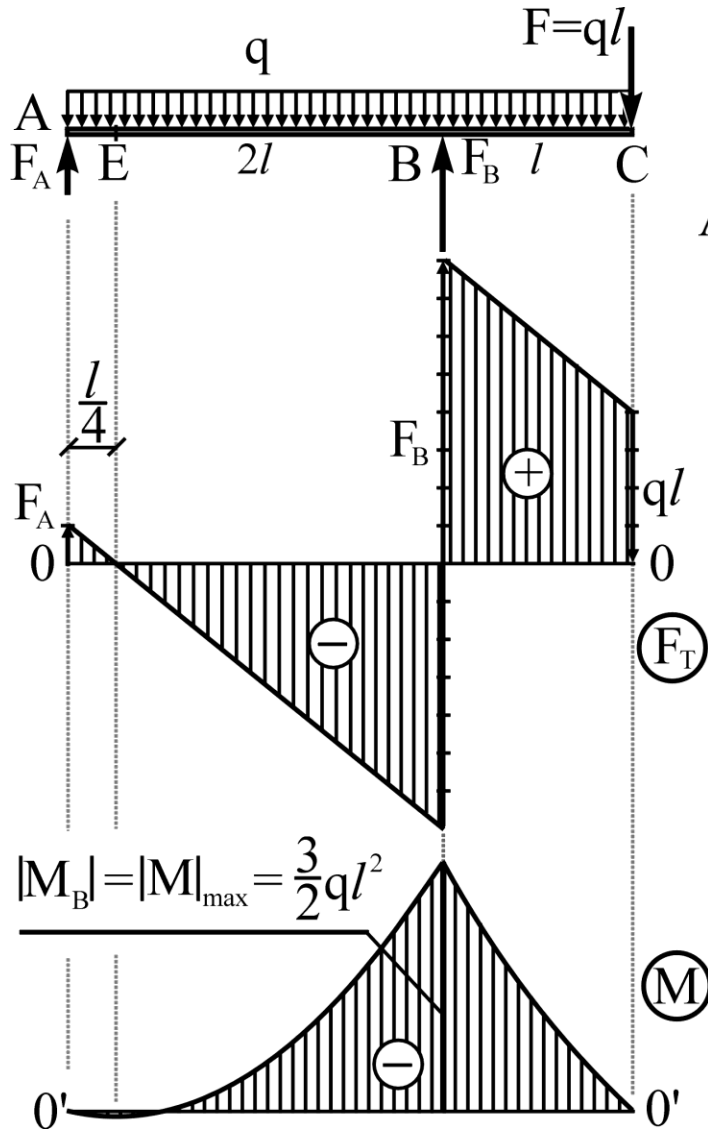
$$I_u = I_u^{(1)} - I_u^{(2)} = \frac{(8c)(4c)^3}{3} - \frac{(3c)^4 \pi}{8} = \left( \frac{512}{3} - \frac{81\pi}{8} \right) c^4 \approx 138,86c^4,$$

$$I_x = I_u - A \cdot |v_T|^2 = \left[ \left( \frac{512}{3} - \frac{81\pi}{8} \right) - \frac{92^2}{2(64 - 9\pi)} \right] c^4 \approx 20,4c^4.$$

**Određivanje otpornog momenta i maksimalnog normalnog napona:**

$$|y|_{\max} = v_T = 2,575c \Rightarrow W_x = \frac{I_x}{|y|_{\max}} = 2,5752c^3 \Rightarrow \sigma_{\max} = \frac{|M|_{\max}}{W_x} = 0,25888 \frac{M}{c^3}.$$

**Primer 3.4** Za zadat gredni nosač: odrediti otpore oslonaca, nacrtati dijagrame momenata savijanja i transverzalnih sila, odrediti maksimalni normalni napon i tangencijalni napon u tački  $N$  preseka  $B+\varepsilon$ ? Poznate veličine su  $q$ ,  $l$  i  $c$ .



**Jednačine ravnoteže:**

$$\sum M_{Ai} = -q \cdot 3l \cdot \frac{3l}{2} + F_B \cdot 2l - ql \cdot 3l = 0$$

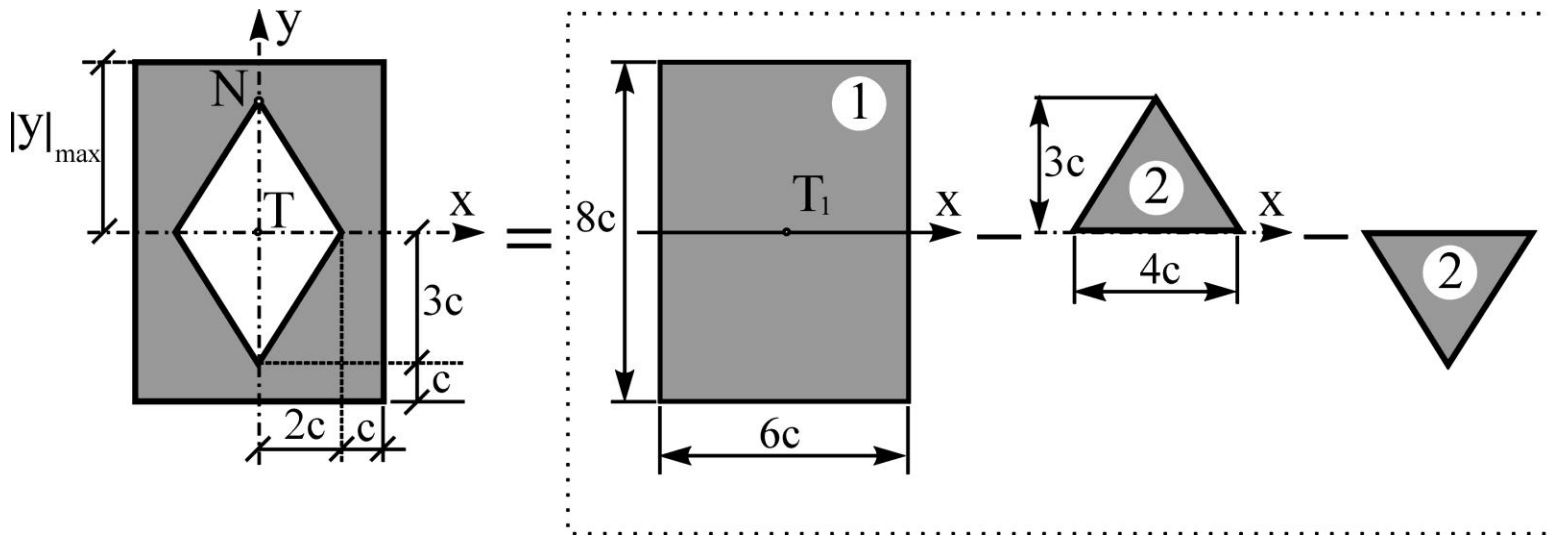
$$\Rightarrow F_B = \frac{15}{4}ql,$$

$$\sum Y_i = F_A - q \cdot 3l + F_B - ql = 0 \Rightarrow F_A = \frac{1}{4}ql.$$

**Napadni momenti :**

$$M_E = \sum M_{Ei}^l = +F_A \cdot \frac{l}{4} - q \cdot \frac{l}{4} \cdot \frac{l}{8} = +\frac{ql^2}{32},$$

$$M_B = \sum M_{Bi}^d = -ql \cdot l - q \cdot l \cdot \frac{l}{2} = -\frac{3}{2}ql^2.$$



**Određivanje momenta inercije za neutralnu osu:**

$$I_x = I_x^{(1)} - 2I_x^{(2)} = \frac{(6c) \cdot (8c)^3}{12} - 2 \frac{(4c) \cdot (3c)^3}{12} = 238c^4.$$

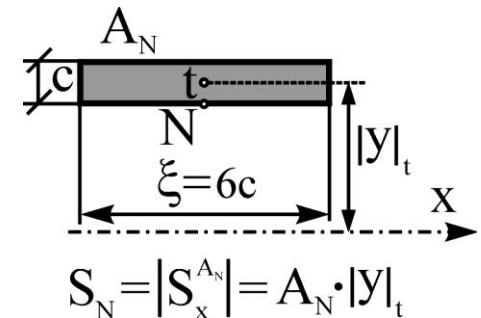
**Određivanje otpornog momenta i maksimalnog normalnog napona:**

$$|y|_{\max} = 4c \Rightarrow W_x = \frac{I_x}{|y|_{\max}} = \frac{119}{2} c^3 \Rightarrow \sigma_{\max} = \frac{|M|_{\max}}{W_x} = \frac{3}{119} \frac{ql^2}{c^3}.$$

**Određivanje tangencijalnog napona:**

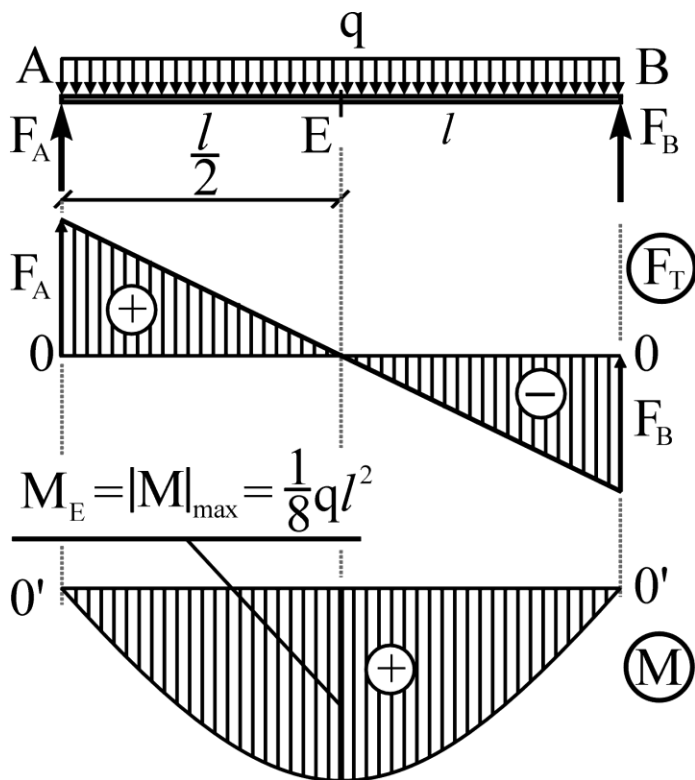
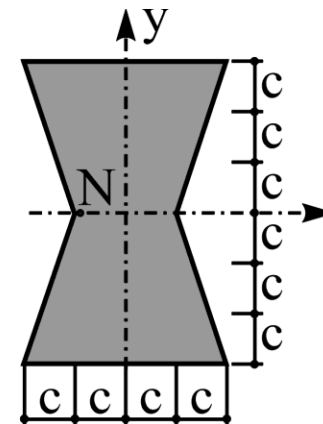
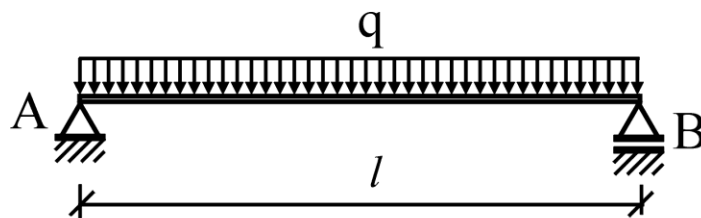
$$|F_T|_{B+\varepsilon} = 2ql, \quad S_N = 6c^2 \cdot \frac{7c}{2} = 21c^3 \Rightarrow$$

$$\tau_N = \frac{|F_T|_{B+\varepsilon} \cdot S_N}{I_x \cdot \xi} = \frac{2ql \cdot 21c^3}{238c^4 \cdot 6c} = \frac{1}{34} \frac{ql}{c^2}.$$





**Primer 3.5** Za zadat gredni nosač: odrediti otpore oslonaca, nacrtati dijagrame momenata savijanja i transverzalnih sila, odrediti maksimalni normalni napon i tangencijalni napon u tački  $N$  preseka  $B$ - $\varepsilon$ ? Poznate veličine su  $q$ ,  $l$  i  $c$ .



**Jednačine ravnoteže:**

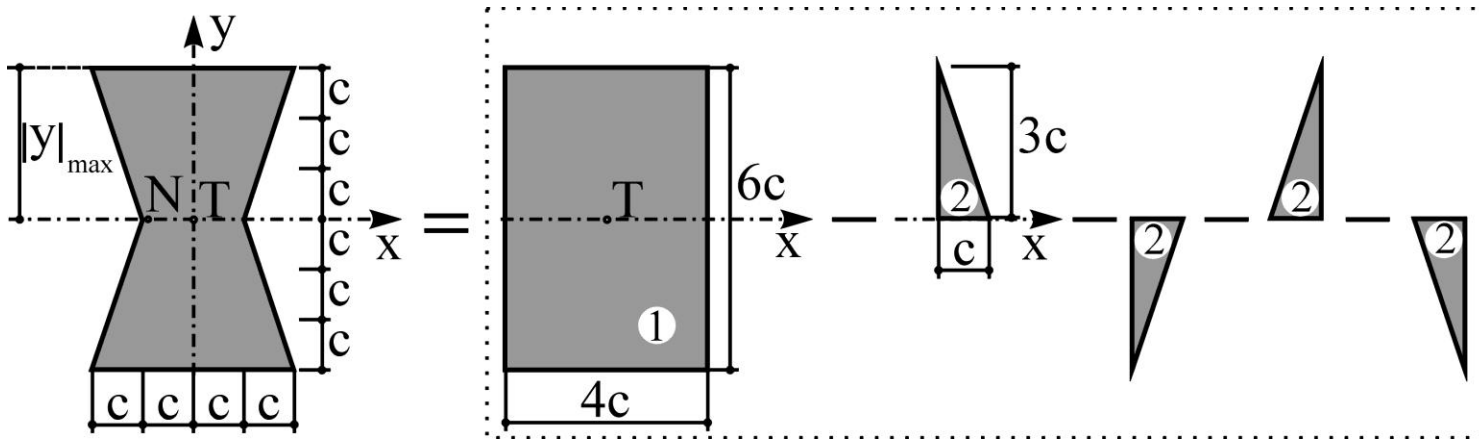
$$\sum M_{Ai} = -q \cdot l \cdot \frac{l}{2} + F_B \cdot l = 0 \Rightarrow F_B = \frac{1}{2} ql,$$

$$\sum Y_i = F_A - q \cdot l + F_B = 0 \Rightarrow F_A = \frac{1}{2} ql.$$

**Napadni momenti :**

Pošto su napadni momenti (momenti savijanja) u tačkama  $A$  i  $B$ , jednaki „0“ izračunajmo njegovu ekstremnu vrednost u tački  $E$ :

$$M_E = \sum M_{Ei}^l = +F_A \cdot \frac{l}{2} - q \cdot \frac{l}{2} \cdot \frac{l}{4} = +\frac{ql^2}{8}.$$



**Određivanje momenta inercije za neutralnu osu:**

$$I_x = I_x^{(1)} - 4I_x^{(2)} = \frac{(4c) \cdot (6c)^3}{12} - 4 \frac{(c) \cdot (3c)^3}{12} = 63c^4.$$

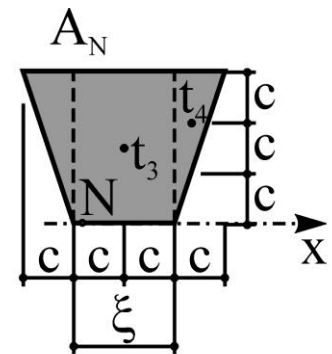
**Određivanje otpornog momenta i maksimalnog normalnog napona:**

$$|y|_{\max} = 3c \Rightarrow W_x = \frac{I_x}{|y|_{\max}} = 21c^3 \Rightarrow \sigma_{\max} = \frac{|M|_{\max}}{W_x} = \frac{1}{168} \frac{ql^2}{c^3}.$$

**Određivanje tangencijalnog napona:**  $|F_T|_{B-\varepsilon} = \frac{ql}{2},$

$$S_N = |S_x^{A_N}| = A_3 \cdot |y|_{t_3} + 2 \cdot A_4 \cdot |y|_{t_4} = 6c^2 \cdot \frac{3c}{2} + 2 \cdot \frac{3c^2}{2} \cdot 2c = 15c^3$$

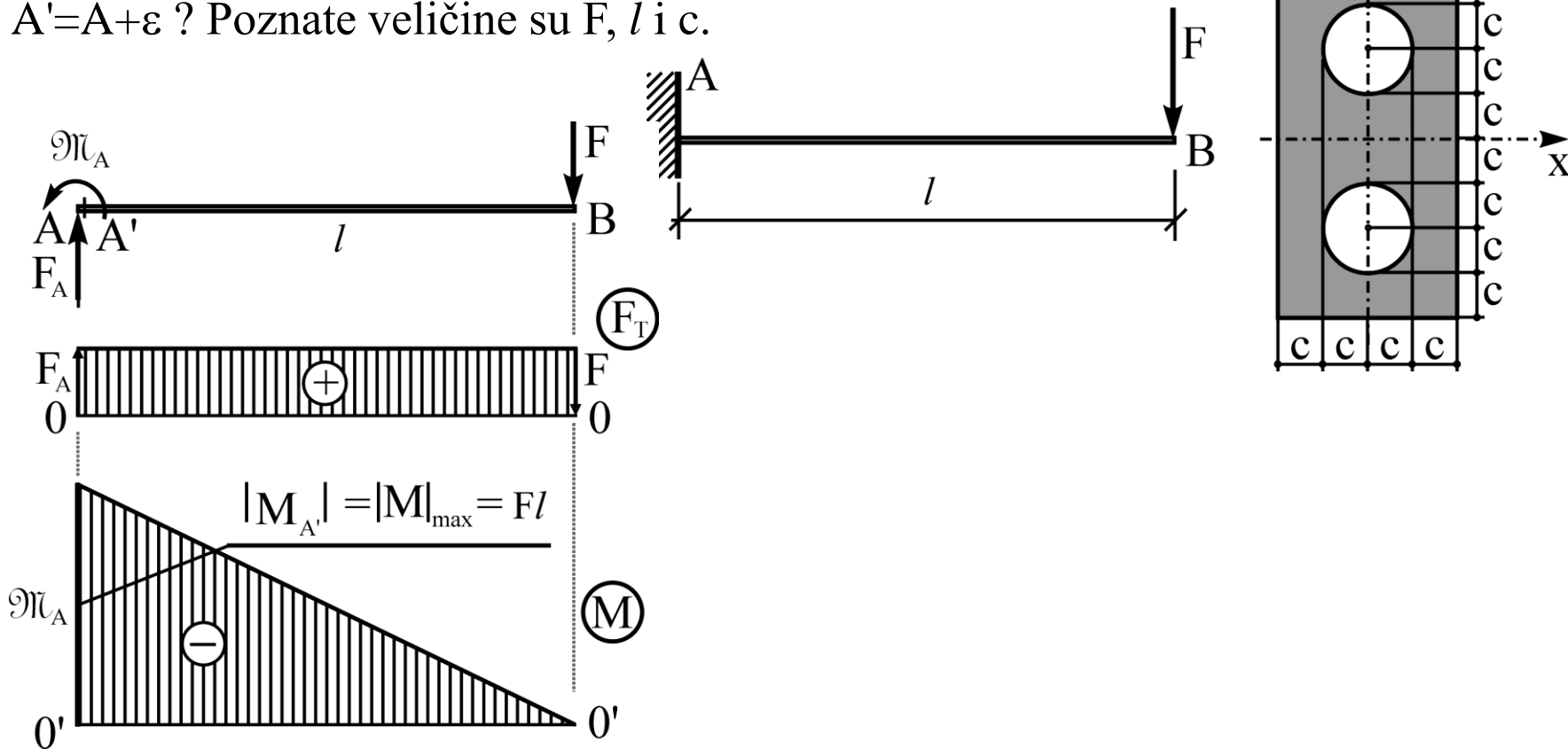
$$\Rightarrow \tau_N = \frac{|F_T|_{B-\varepsilon} \cdot S_N}{I_x \cdot \xi} = \frac{\frac{ql}{2} \cdot 15c^3}{63c^4 \cdot 2c} = \frac{5}{84} \frac{ql}{c^2}.$$



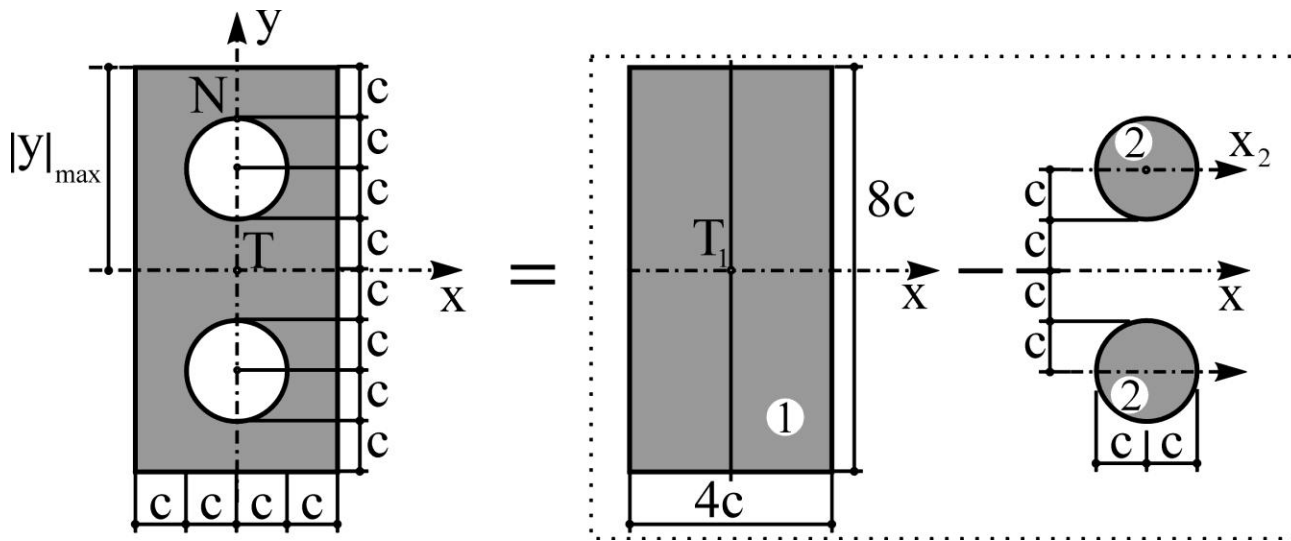
$$S_N = |S_x^{A_N}| = A_3 \cdot |y|_{t_3} + 2A_4 \cdot |y|_{t_4}$$

**Primer 3.6** Za zadatau konzolu: odrediti reakcije u uklještenju, nacrtati dijagrame momenata savijanja i transverzalnih sila, odrediti maksimalni normalni napon i tangencijalni napon u tački N preseka

$A'=A+\varepsilon$  ? Poznate veličine su  $F$ ,  $l$  i  $c$ .



Pošto je napadni moment (moment savijanja) u tački  $B$  jednak „0“, izračunajmo njegovu vrednost samo u tački  $A'$ , koja se nalazi beskonačno blizu desno od tačke  $A$ , u kojoj dejstvuje moment uklještenja  $M_A$  :  $M_{A'} = \sum M_{A'i}^l = -M_A = -Fl$ .



### Određivanje momenta inercije za neutralnu osu:

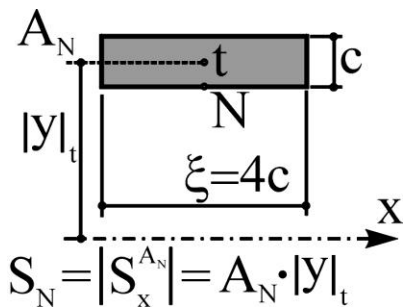
$$I_x = I_x^{(1)} - 2I_x^{(2)} = I_x^{(1)} - 2\left[I_{x_2}^{(2)} + A_2 \cdot (2c)^2\right] = \frac{(4c) \cdot (8c)^3}{12} - 2\left[\frac{c^4 \cdot \pi}{4} + c^2 \pi \cdot (2c)^2\right],$$

$$\Rightarrow I_x = B \cdot c^4, \quad B \approx 143,96.$$

### Određivanje otpornog momenta i maksimalnog normalnog napona:

$$|y|_{\max} = 4c \Rightarrow W_x = \frac{I_x}{|y|_{\max}} = \frac{B}{4} c^3 \Rightarrow \sigma_{\max} = \frac{|M|_{\max}}{W_x} = \frac{4 Fl}{B c^3} \approx \frac{2,7785 Fl}{100 c^3}.$$

### Određivanje tangencijalnog napona:

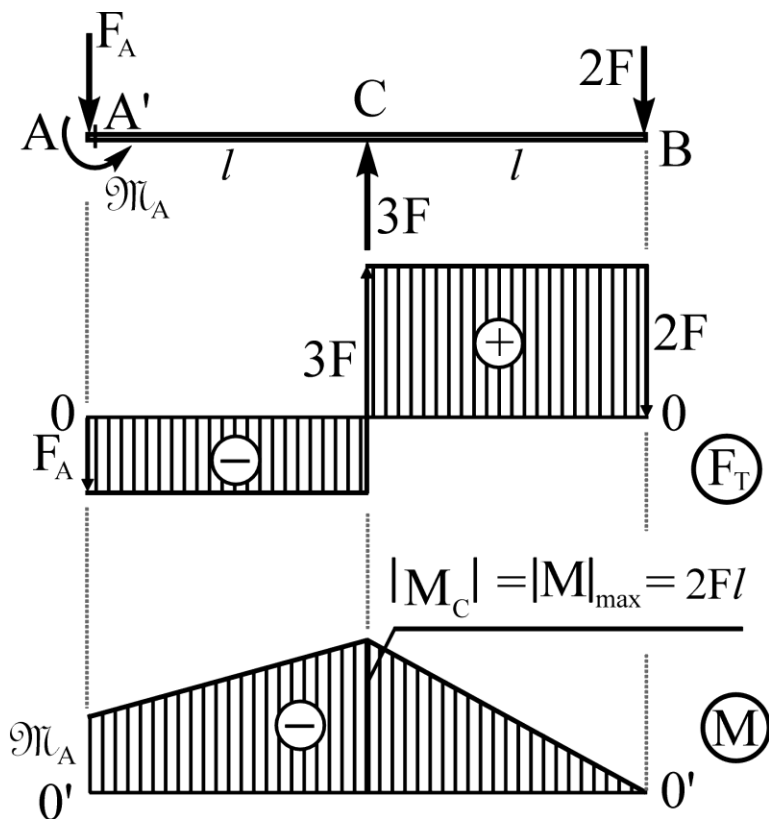
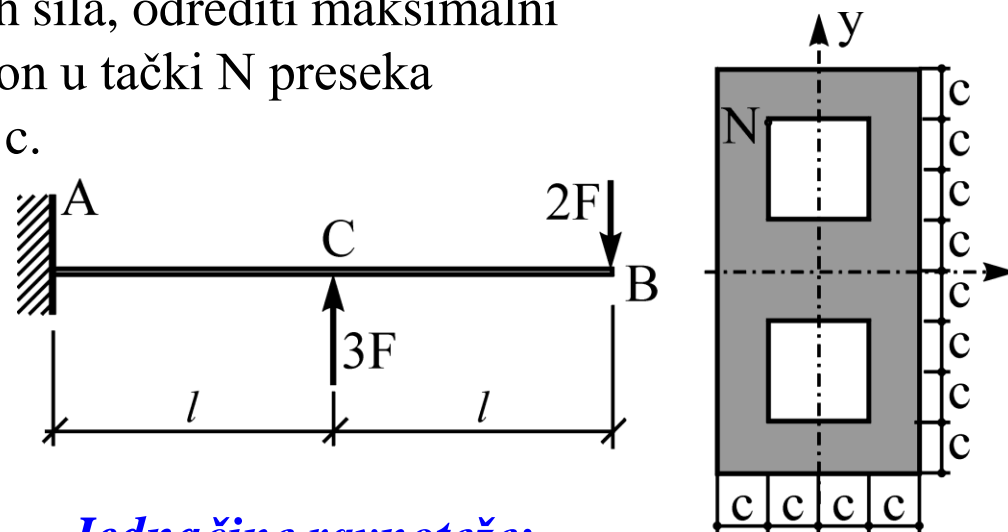


$$|F_T|_{A+\epsilon} = F, \quad S_N = 4c^2 \cdot \frac{7c}{2} = 14c^3 \Rightarrow$$

$$\tau_N = \frac{|F_T|_{A+\epsilon} \cdot S_N}{I_x \cdot \xi} = \frac{F \cdot 14c^3}{Bc^4 \cdot 4c} \approx \frac{2,4312 F}{100 c^2}.$$

**Primer 3.7** Za zadanu konzolu: odrediti reakcije u uklještenju, nacrtati dijagrame momenata savijanja i transverzalnih sila, odrediti maksimalni normalni napon i tangencijalni napon u tački N preseka

$A' = A + \varepsilon$  ? Poznate veličine su  $F$ ,  $l$  i  $c$ .



**Jednačine ravnoteže:**

$$\sum M_{Ai} = +M_A + 3F \cdot l - 2F \cdot 2l = 0$$

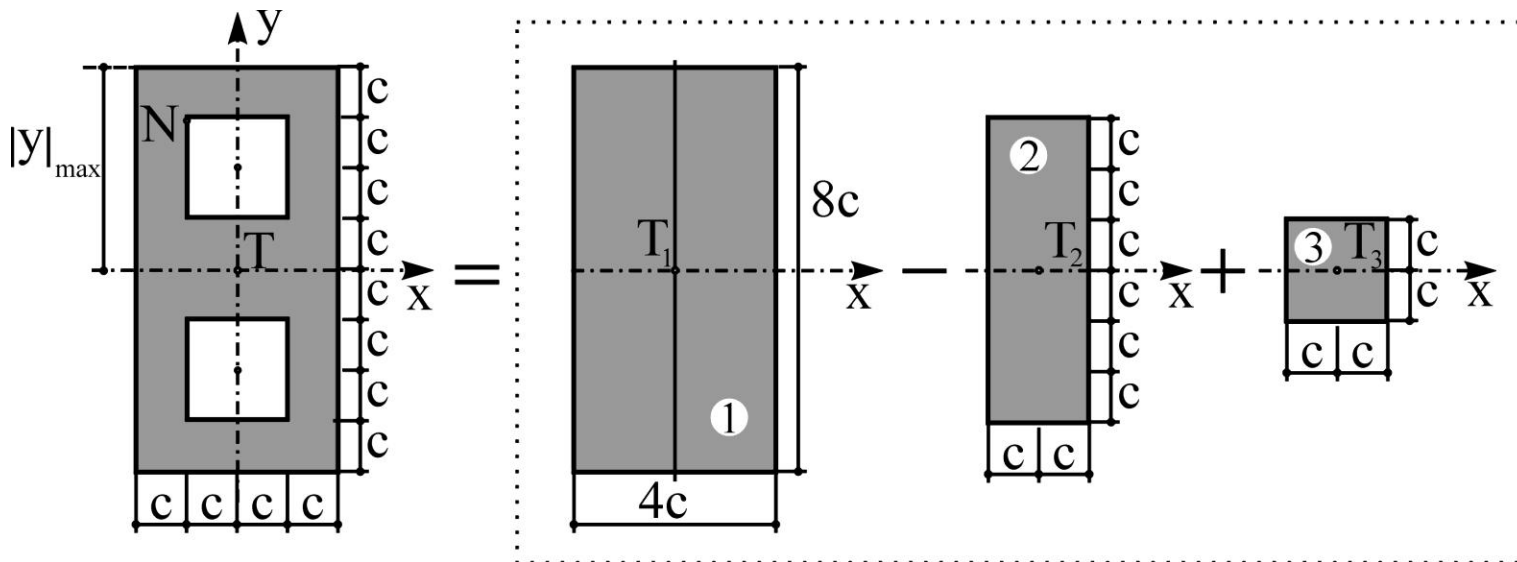
$$\Rightarrow M_A = F \cdot l,$$

$$\sum Y_i = -F_A + 3F - 2F = 0 \Rightarrow F_A = F.$$

**Napadni momenti :**

$$M_{A'} = \sum M_{A'i}^l = -M_A = -Fl,$$

$$M_C = \sum M_{Ci}^l = -M_A - F_A \cdot l = -2Fl.$$



**Određivanje momenta inercije za neutralnu osu:**

$$I_x = I_x^{(1)} - I_x^{(2)} + I_x^{(3)} = \frac{(4c) \cdot (8c)^3}{12} - \frac{(2c) \cdot (6c)^3}{12} + \frac{(2c) \cdot (2c)^3}{12} = 136c^4.$$

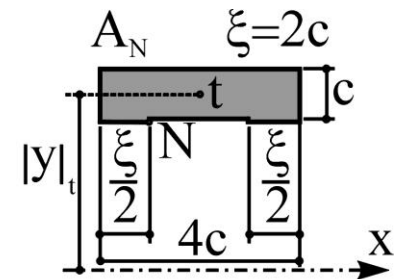
**Određivanje otpornog momenta i maksimalnog normalnog napona:**

$$|y|_{\max} = 4c \Rightarrow W_x = \frac{I_x}{|y|_{\max}} = 34c^3 \Rightarrow \sigma_{\max} = \frac{|M|_{\max}}{W_x} = \frac{1}{17} \frac{Fl}{c^3}.$$

**Određivanje tangencijalnog napona:**

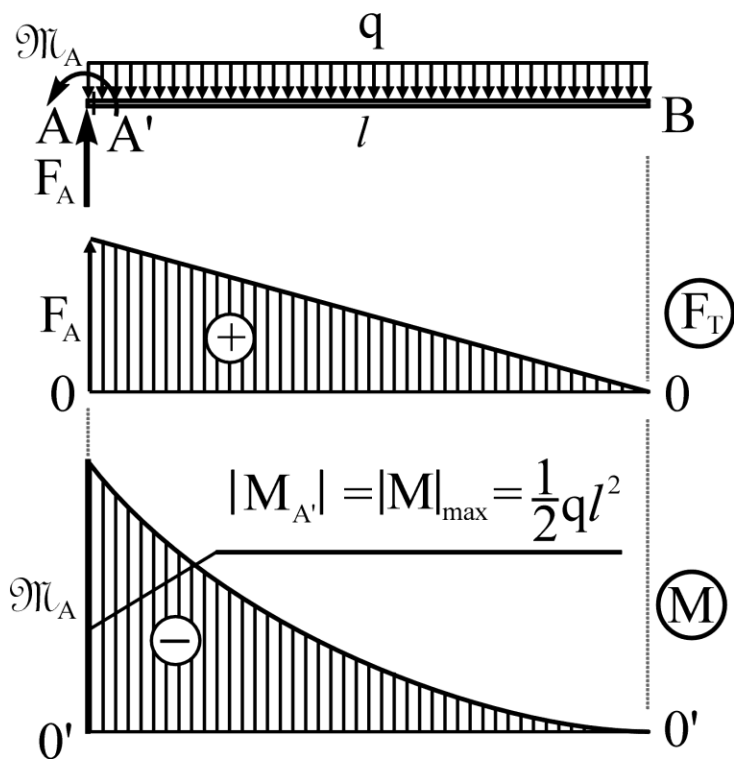
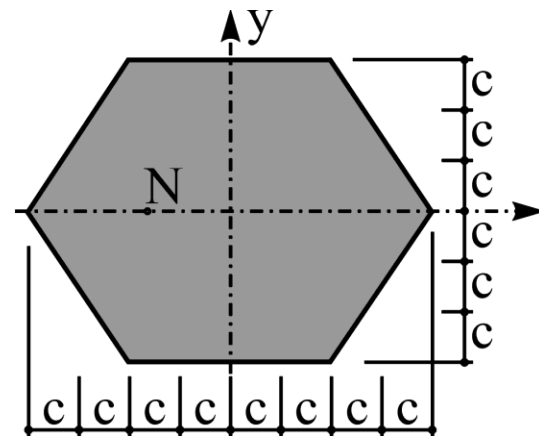
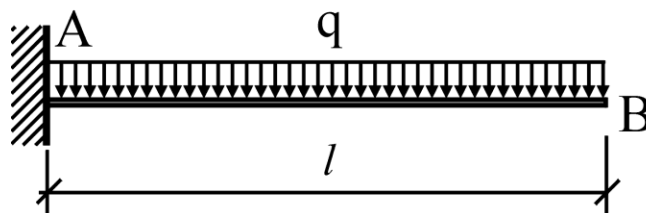
$$|F_T|_{A+\varepsilon} = F, \quad S_N = 4c^2 \cdot \frac{7c}{2} = 14c^3 \Rightarrow$$

$$\tau_N = \frac{|F_T|_{A+\varepsilon} \cdot S_N}{I_x \cdot \xi} = \frac{F \cdot 14c^3}{136c^4 \cdot 2c} = \frac{7}{136} \frac{F}{c^2}.$$



$$S_N = |S_x^{A_N}| = A_N \cdot |y|_t$$

**Primer 3.8** Za zadatau konzolu: odrediti reakcije u uklještenju, nacrtati dijagrame momenata savijanja i transverzalnih sila, odrediti maksimalni normalni napon i tangencijalni napon u tački N preseka  $A'=A+\varepsilon$ ? Poznate veličine su  $q$ ,  $l$  i  $c$ .



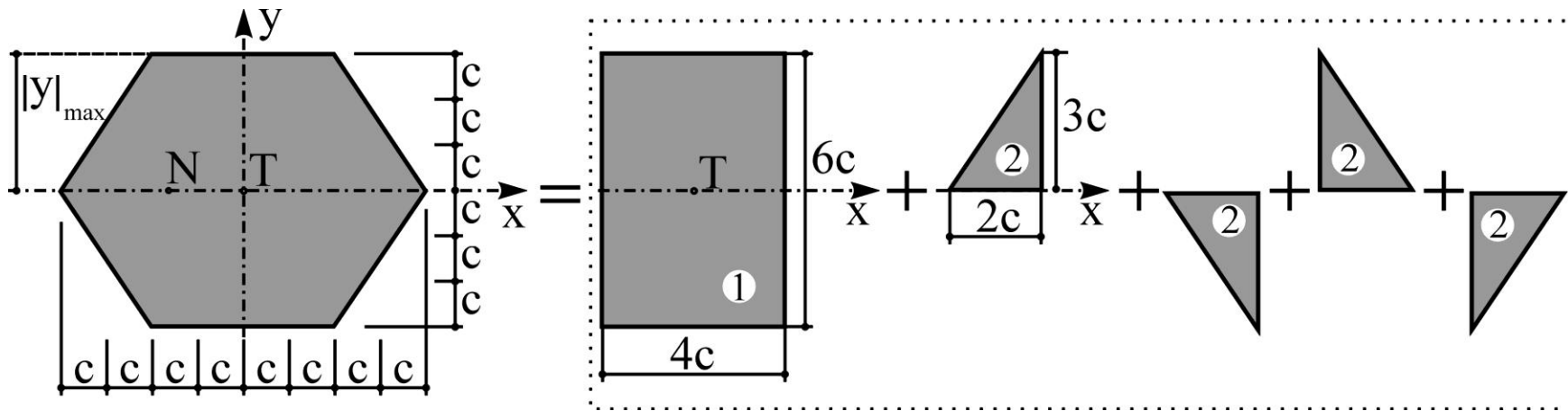
*Jednačine ravnoteže:*

$$\sum M_{Ai} = +M_A + q \cdot l \cdot \frac{l}{2} = 0 \Rightarrow M_A = \frac{1}{2}ql^2,$$

$$\sum Y_i = +F_A + q \cdot l = 0 \Rightarrow F_A = ql.$$

*Napadni momenti :*

$$M_{A'} = \sum M_{A'i}^l = -M_A = -\frac{1}{2}ql^2.$$

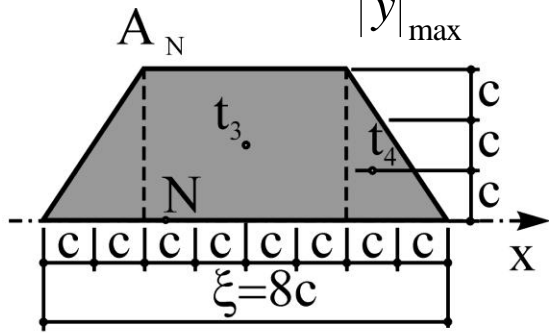


**Određivanje momenta inercije za neutralnu osu:**

$$I_x = I_x^{(1)} + 4I_x^{(2)} = \frac{(4c) \cdot (6c)^3}{12} + 4 \frac{(2c) \cdot (3c)^3}{12} = 90c^4.$$

**Određivanje otpornog momenta i maksimalnog normalnog napona:**

$$|y|_{\max} = 3c \Rightarrow W_x = \frac{I_x}{|y|_{\max}} = 30c^3 \Rightarrow \sigma_{\max} = \frac{|M|_{\max}}{W_x} = \frac{1}{60} \frac{ql^2}{c^3}.$$



**Određivanje tangencijalnog napona:**

$$|F_T|_{A+\varepsilon} = ql, \quad S_N = 12c^2 \cdot \frac{3c}{2} + 2 \cdot 3c^2 \cdot c = 24c^3 \Rightarrow$$

$$\tau_N = \frac{|F_T|_{A+\varepsilon} \cdot S_N}{I_x \cdot \xi} = \frac{ql \cdot 24c^3}{90c^4 \cdot 8c} = \frac{1}{30} \frac{ql}{c^2}.$$

$$S_N = |S_x^{A_N}| = A_3 \cdot |y|_{t_3} + 2A_4 \cdot |y|_{t_4}$$