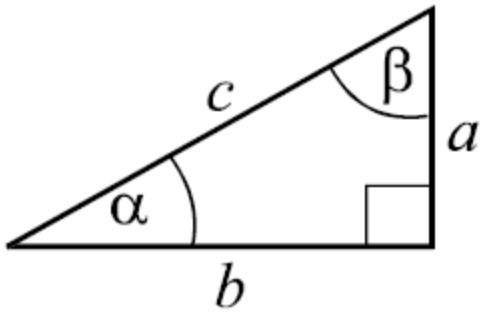
Mehanika
Matematička priprema

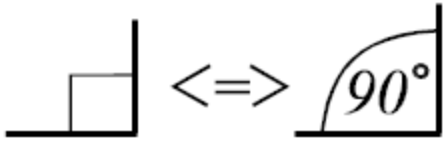
D. Radomirović, M. Zuković

Novi Sad, 2022.

PRAVOUGLI TROUGAO



$$\alpha + \beta = 90^0$$



c - hipotenuza
a i *b* - katete

Za ugao α: *b* je nalegla kateta, *a* je naspramna kateta.
Za ugao β : *a* je nalegla kateta, *b* je naspramna kateta.

$$\sin\alpha = \frac{\text{naspramna kateta}}{\text{hipotenuza}} = \frac{a}{c}$$

$$\cos\alpha = \frac{\text{nalegla kateta}}{\text{hipotenuza}} = \frac{b}{c}$$

$$\tan\alpha = \frac{\text{naspramna kateta}}{\text{nalegla kateta}} = \frac{a}{b}$$

$$\cot\alpha = \frac{\text{nalegla kateta}}{\text{naspramna kateta}} = \frac{b}{a}$$

Pitagorina teorema

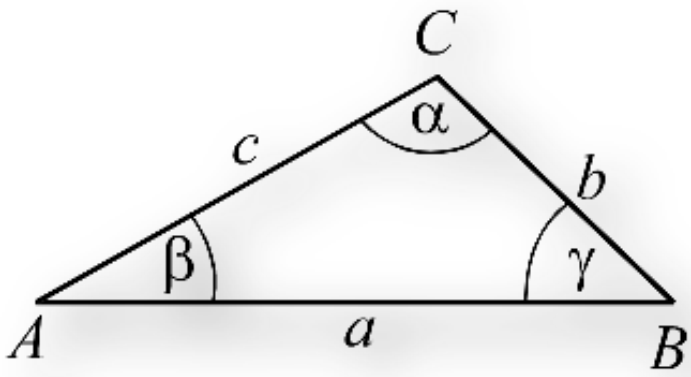
$$a^2 + b^2 = c^2$$

$$\tan x = \frac{\sin x}{\cos x}, \cot x = \frac{\cos x}{\sin x}, \tan x = \frac{1}{\cot x}$$

$$\sin^2 x + \cos^2 x = 1$$

NEJEDNAKOSTRANIČNI TROUGAO

$\alpha+\beta+\gamma=180^0$ $\gamma=180^0-(\alpha+\beta)$



Sinusna teorema

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

Tri jednačine, ali su dve nezavisne

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta},\quad \frac{a}{\sin\alpha}=\frac{c}{\sin\gamma},\quad \frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$$

Kosinusna teorema

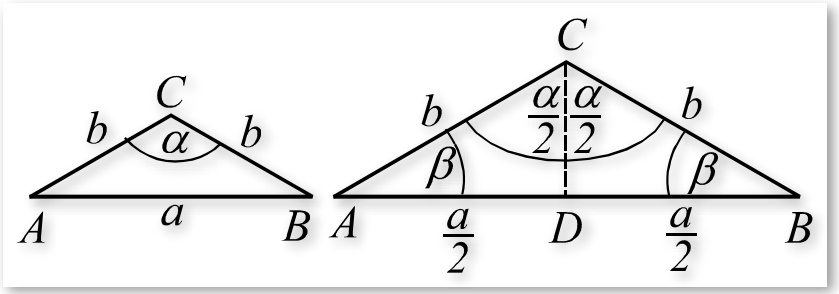
$$c^2=a^2+b^2-2ab\cos\gamma$$

$$a^2=b^2+c^2-2bc\cos\alpha$$

$$b^2=c^2+a^2-2ca\cos\beta$$

$$\Rightarrow \cos\gamma=\frac{a^2+b^2-c^2}{2ab}\quad \text{itd.}$$

JEDNAKOKRAKI TROUGAO

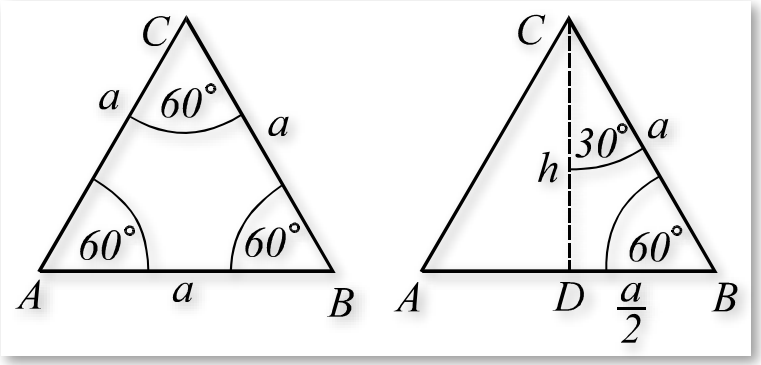


$$\overline{CD} = b\cos\frac{\alpha}{2}$$

$$\frac{a}{2} = b\sin\frac{\alpha}{2} \qquad \frac{a}{2} = b\cos\beta$$

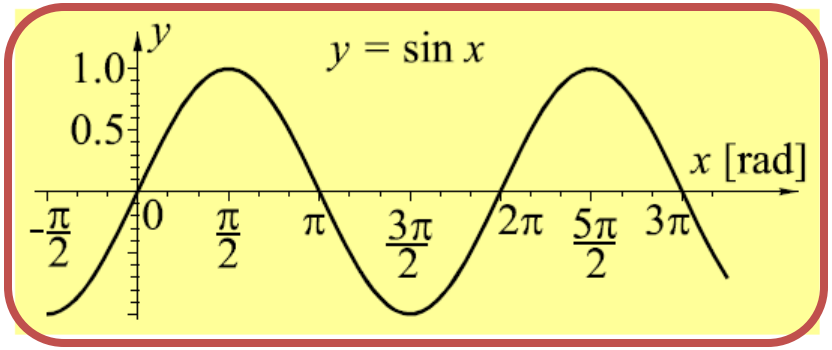
$$\overline{CD} = b\sin\beta$$

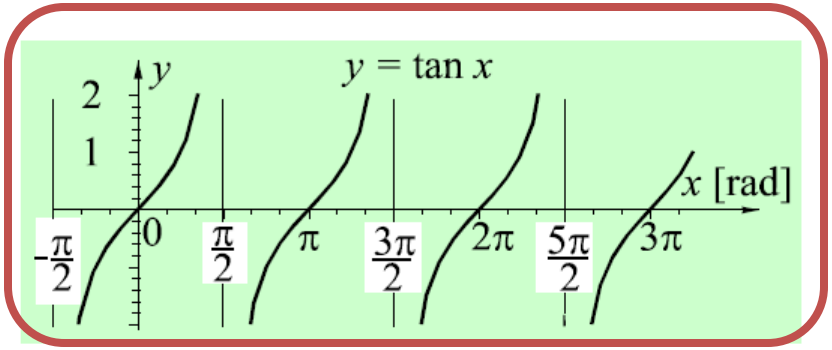
JEDNAKOSTRANIČNI TROUGAO

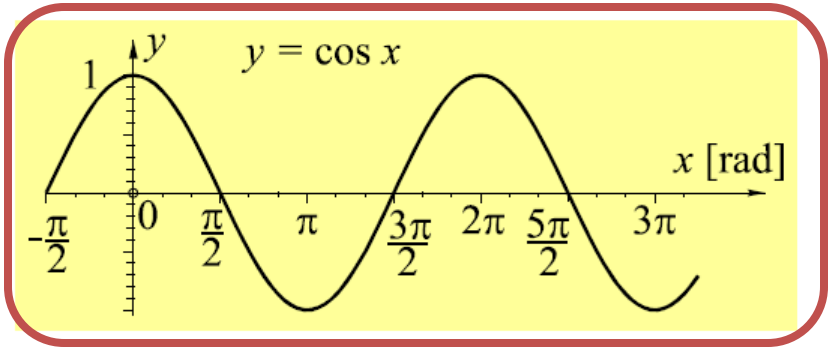


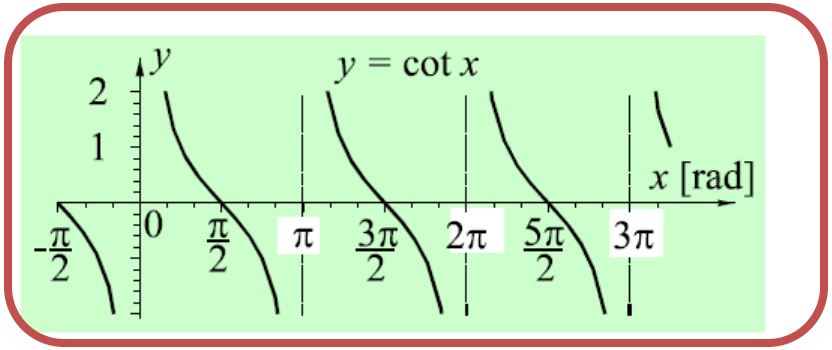
$$h = a\sin 60^0 = a\cos 30^0 = \sqrt{a^2 - \left(\frac{a}{2}\right)^2}$$

GRAFICI TRIGONOMETRIJSKIH FUNKCIJA









$\sin x = \sin(2\pi+x)$ i $\cos x = \cos(2\pi+x)$.

Periodične funkcije perioda 2π

$\tan x = \tan(\pi+x)$ i $\cot x = \cot(\pi+x)$

Periodične funkcije perioda π

ADICIONE FORMULE

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

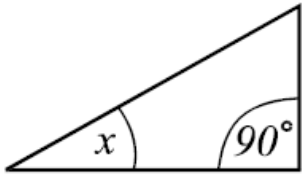
Specijalan slučaj za $\beta=\alpha$

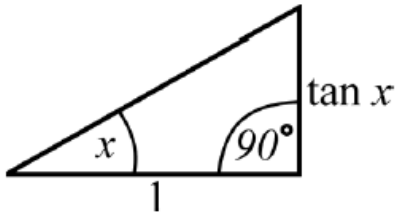
$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

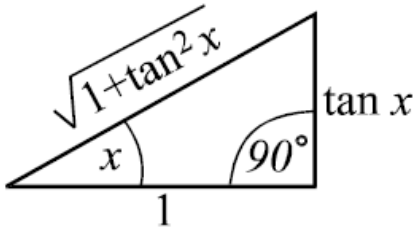
$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

VEZE IZMEĐU TRIGONOMETRIJSKIH FUNKCIJA

Primer 2.2. tan x je poznat, u zavisnosti od tan x, odrediti sin x, cos x i cot x?

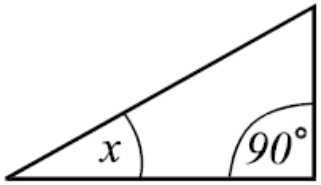


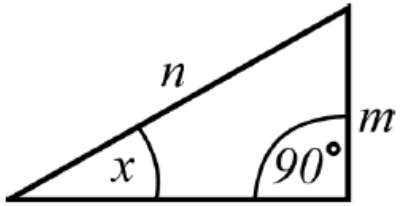


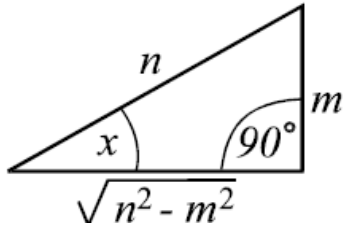


$$\sin x = \frac{\tan x}{\sqrt{1+\tan^2 x}}, \quad \cos x = \frac{1}{\sqrt{1+\tan^2 x}} \quad \cot x = \frac{1}{\tan x}.$$

Primer 2.3 Neka je sin$x$ = m/n, koliki su tan x, cos x i cot x, u zavisnosti od m i n?







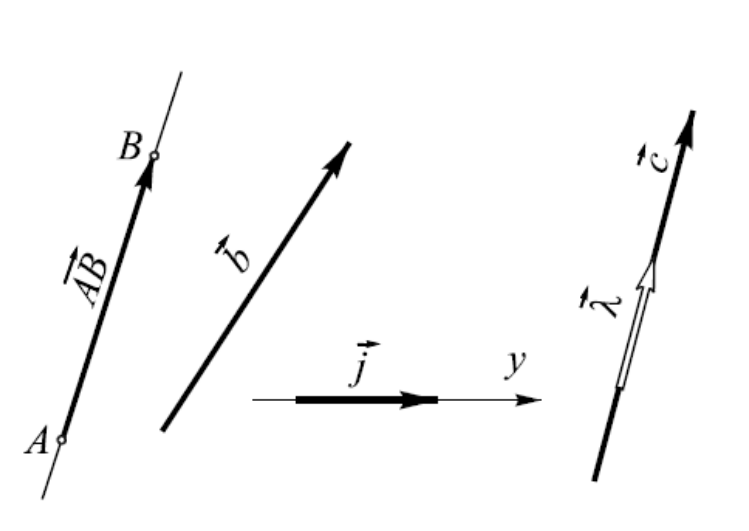
$$\tan x = \frac{m}{\sqrt{n^2 - m^2}}$$

VEKTORI I SKALARI

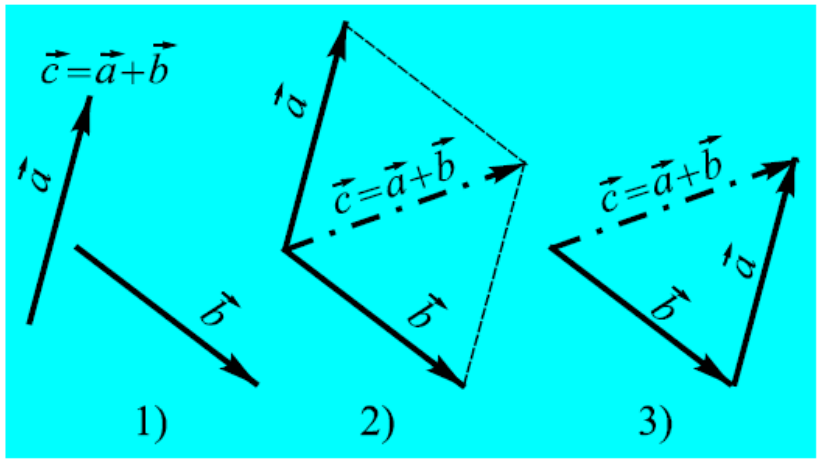
$\vec{j}$, $\vec{\lambda}$ - jedinični vektori ($|\vec{j}| = |\vec{\lambda}| = 1$)

$|\vec{b}| = b$, $|\vec{c}| = c$, $|\overrightarrow{AB}| = \overline{AB}$ - intenziteti vektora

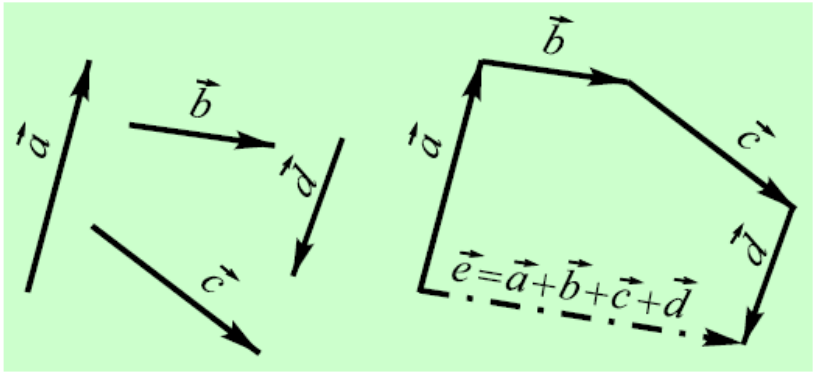
$$\vec{c} = c\vec{\lambda}$$



GRAFIČKI PRIKAZ SABIRANJA VEKTORA

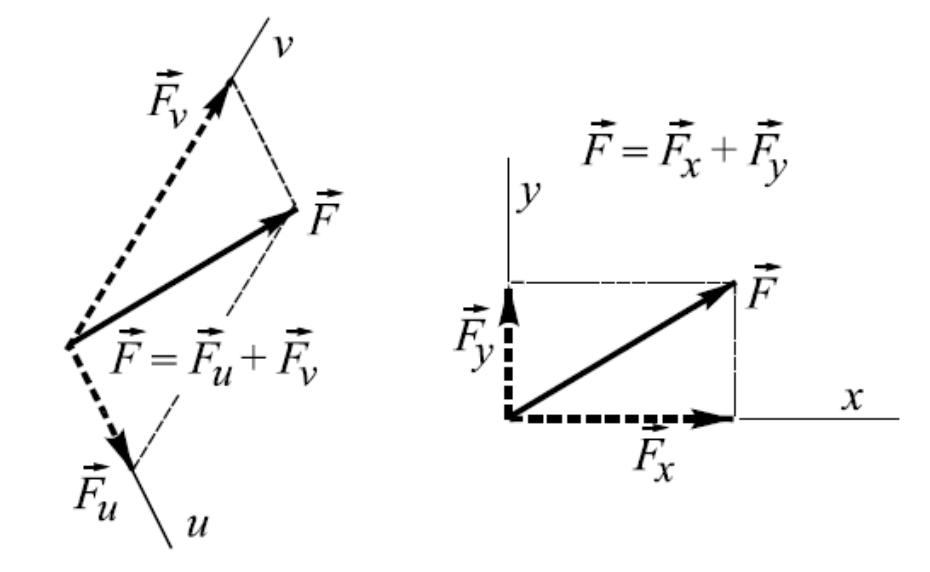


Grafičko sabiranje dva vektora



Grafičko sabiranje više vektora

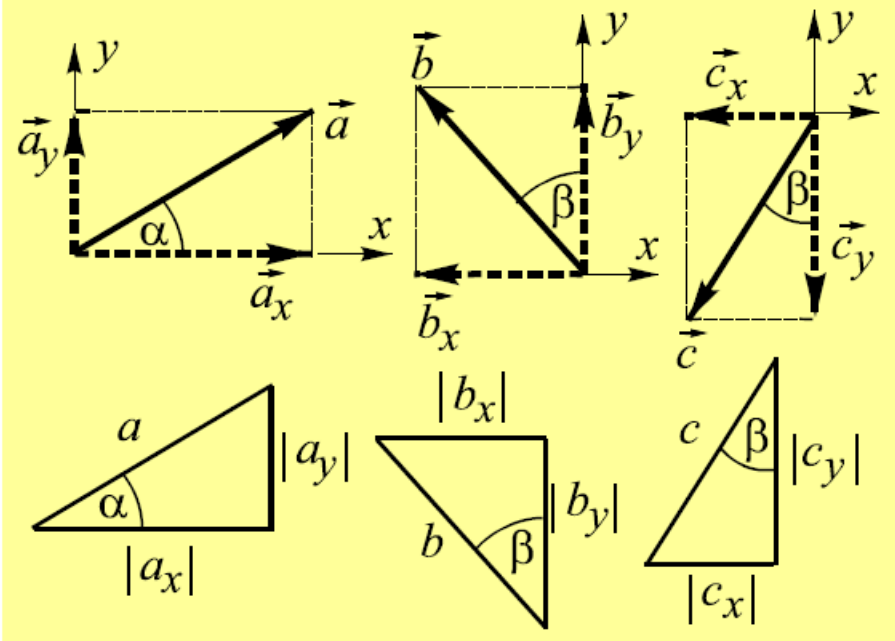
RAZLAGANJE VEKTORA NA DVE KOMPONENTE



$\vec{F}_x$, $\vec{F}_y$ - Komponente vektora $\vec{F}$ u x i y pravcu

Vektor se razlaže preko pravila paralelograma

PROJEKCIJA VEKTORA NA OSU



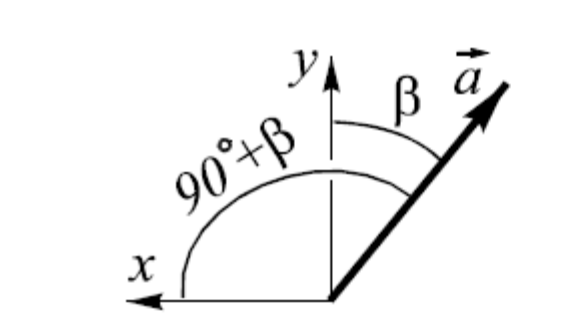
$$a_x = +|a_x| = a\cos\alpha, \quad a_y = +|a_y| = a\sin\alpha$$

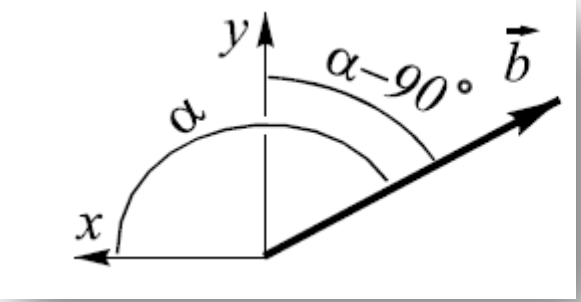
$$b_x = -|b_x| = -b\sin\beta, \quad b_y = +|b_y| = b\cos\beta$$

$$c_x = -|c_x| = -c\sin\beta, \quad c_y = -|c_y| = -c\cos\beta$$

Projekcija vektora na osu je SKALAR

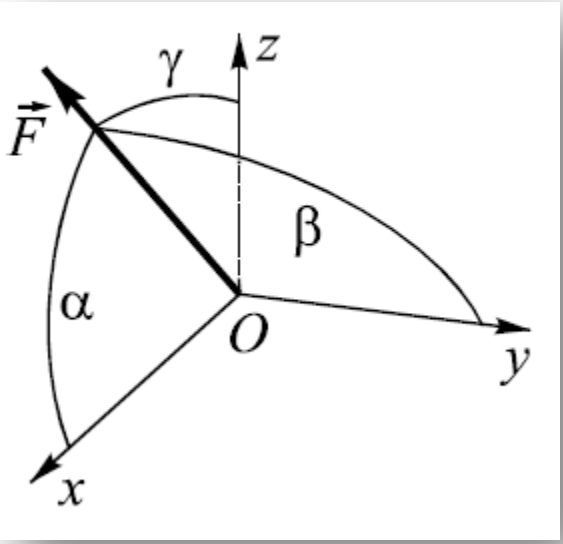
Projekcija vektora na osu za zadat ugao između vektora i pozitivnog dela ose





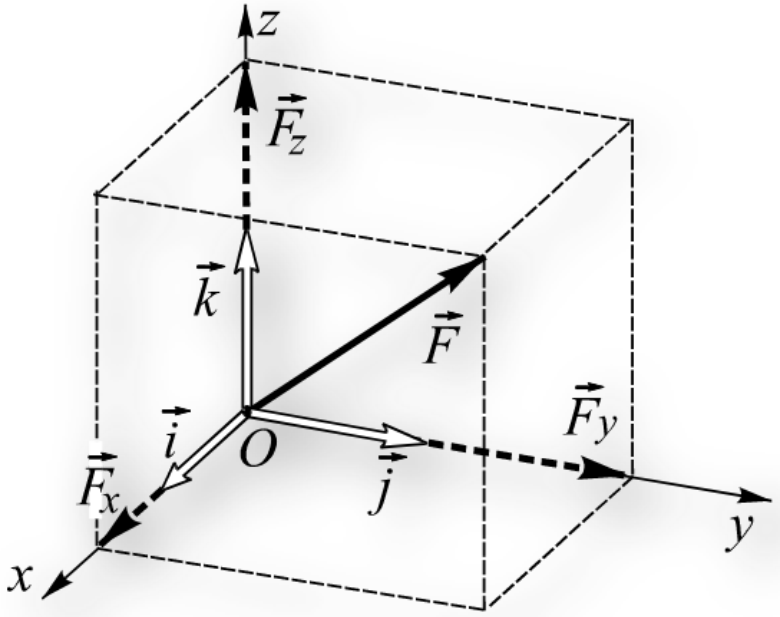
$$a_x = a\cos(90^0 + \beta), \quad a_y = a\cos\beta$$

$$b_x = b\cos\alpha, \quad b_y = b\cos(\alpha - 90^0)$$



$$F_x = F\cos\alpha, \quad F_y = F\cos\beta, \quad F_z = F\cos\gamma$$

VEKTORI IZRAŽENI PREKO ORTOGONALNIH JEDINIČNIH VEKTORA



$$\vec{F} = \vec{F}_x + \vec{F}_y + \vec{F}_z$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$\vec{F}_x = F_x\vec{i}$$

$$\vec{F}_y = F_y\vec{j}$$

$$\vec{F}_z = F_z\vec{k}$$

$$\vec{a}+\vec{b}=\left(a_x\vec{i}+a_y\vec{j}+a_z\vec{k}\right)+\left(b_x\vec{i}+b_y\vec{j}+b_z\vec{k}\right)=$$
$$=\left(a_x+b_x\right)\vec{i}+\left(a_y+b_y\right)\vec{j}+\left(a_z+b_z\right)\vec{k},$$

$$\vec{a}_R=\sum\vec{a}_i=\sum\left(a_{ix}\vec{i}+a_{iy}\vec{j}+a_{iz}\vec{k}\right)=$$
$$=\left(\sum a_{ix}\right)\vec{i}+\left(\sum a_{iy}\right)\vec{j}+\left(\sum a_{iz}\right)\vec{k}$$
$$\Rightarrow a_{Rx}=\sum a_{ix},\ a_{Ry}=\sum a_{iy},\ a_{Rz}=\sum a_{iz},$$

$$\vec{a}-\vec{b}=\left(a_x\vec{i}+a_y\vec{j}+a_z\vec{k}\right)-\left(b_x\vec{i}+b_y\vec{j}+b_z\vec{k}\right)=$$
$$=\left(a_x-b_x\right)\vec{i}+\left(a_y-b_y\right)\vec{j}+\left(a_z-b_z\right)\vec{k}.$$

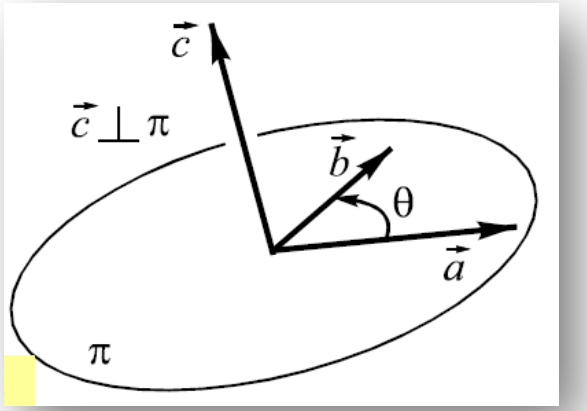
SKALARNI I VEKTORSKI PROIZVOD

- Skalarni proizvod

$$\vec{a} \cdot \vec{b} = ab\cos\theta$$

- Vektorski proizvod

$$\vec{c} = \vec{a} \times \vec{b}$$
$$c = ab\sin\theta$$



Ravan π obrazuju vektori koji se vektorski množe
Vektor je upravan na ravan π, a smer mu je određen pravilom desne ruke

Primer

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$
$$\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$$

$$\vec{i}\cdot\vec{j} = 1\cdot 1\cdot\cos 90^0 = 0$$
$$\vec{i}\cdot\vec{i} = 1\cdot 1\cdot\cos 0 = 1$$

$$\vec{a}\cdot\vec{b} = \left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)\cdot\left(b_x\vec{i} + b_y\vec{j} + b_z\vec{k}\right) =$$
$$= a_xb_x\vec{i}\cdot\vec{i} + a_xb_y\vec{i}\cdot\vec{j} + \ldots = a_xb_x + a_yb_y + a_zb_z.$$

$$\vec{i}\times\vec{i} = 0 \qquad \left|\vec{i}\times\vec{i}\right| = 1\cdot 1\cdot\sin 0^0 = 0$$
$$\vec{i}\times\vec{k} = -\vec{j} \qquad \left|\vec{i}\times\vec{k}\right| = 1\cdot 1\cdot\sin 90^0 = 1$$
$$2\vec{k}\times 3\vec{i} = 6\vec{j} \qquad \left|2\vec{k}\times 3\vec{i}\right| = 2\cdot 3\cdot\sin 90^0 = 6$$

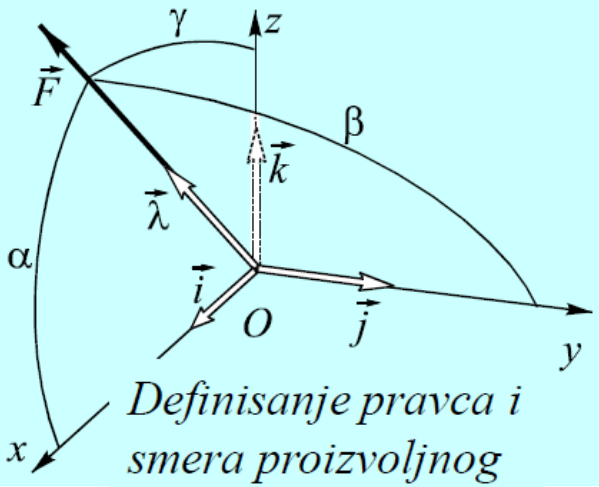
$$\vec{a}\times\vec{b} = \left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)\times\left(b_x\vec{i} + b_y\vec{j} + b_z\vec{k}\right) =$$
$$= a_xb_x\vec{i}\times\vec{i} + a_xb_y\vec{i}\times\vec{j} + a_xb_z\vec{i}\times\vec{k} + \ldots =$$
$$= \left(a_yb_z - a_zb_y\right)\vec{i} + \left(a_zb_x - a_xb_z\right)\vec{j} +$$
$$\left(a_xb_y - a_yb_x\right)\vec{k}$$

Jednostavnije određivanje vektorskog proizvoda $\vec{a}\times\vec{b}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\vec{i}\begin{vmatrix}a_y & a_z\\ b_y & b_z\end{vmatrix}-\vec{j}\begin{vmatrix}a_x & a_z\\ b_x & b_z\end{vmatrix}+\vec{k}\begin{vmatrix}a_x & a_y\\ b_x & b_y\end{vmatrix}=$$

$$=\vec{i}\left(a_yb_z-a_zb_y\right)-\vec{j}\left(a_xb_z-a_zb_x\right)+\vec{k}\left(a_xb_y-a_yb_x\right)=$$

$$=\left(a_yb_z-a_zb_y\right)\vec{i}+\left(a_zb_x-a_xb_z\right)\vec{j}+\left(a_xb_y-a_yb_x\right)\vec{k}$$



Definisanje pravca i smera proizvoljnog vektora u prostoru

$$\vec{\lambda}=\cos\alpha\,\vec{i}+\cos\beta\,\vec{j}+\cos\gamma\,\vec{k}$$

$$\vec{\lambda}\cdot\vec{\lambda}=\left(\cos\alpha\,\vec{i}+\cos\beta\,\vec{j}+\cos\gamma\,\vec{k}\right)\cdot\left(\cos\alpha\,\vec{i}+\cos\beta\,\vec{j}+\cos\gamma\,\vec{k}\right)\Rightarrow$$

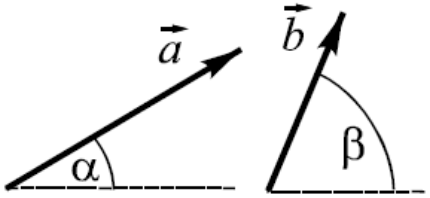
$$1=\cos^2\alpha+\cos^2\beta+\cos^2\gamma$$

$$\vec{F}=X\,\vec{i}+Y\,\vec{j}+Z\,\vec{k}$$

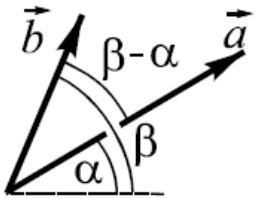
$$F^2=X^2+Y^2+Z^2\Rightarrow F=\sqrt{X^2+Y^2+Z^2}$$

Primer 2.7 Da li je projekcija ma kog vektora na neku osu jednaka skalarnom proizvodu tog vektora i jediničnog vektora te ose?

Primer 2.8



1)



2)

Ako su poznati vektori $\vec{a}$ i $\vec{b}$ preko svojih intenziteta i uglova α i β, primenom skalarnog proizvoda odrediti intenzitet vektora $\vec{c}$ koji predstavlja zbir vektora $\vec{a}$ i $\vec{b}$

$$\vec{c}\cdot\vec{c}=\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=\vec{a}\cdot\vec{a}+\vec{b}\cdot\vec{b}+2\vec{a}\cdot\vec{b}\Rightarrow c^2=a^2+b^2+2ab\cos(\beta-\alpha)$$

$$c=\sqrt{a^2+b^2+2ab\cos(\beta-\alpha)}$$

Primer 2.9

Dokazati da se vektorskim množenjem paralelnih vektora dobija nula vektor?

Primer 2.10

Dokazati da je mešoviti proizvod $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}$ jednak nuli ako je vektor $\vec{c}$ istog pravca kao vektor $\vec{a}$ ili vektor $\vec{b}$?

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