

## DIFERENCIJALNE JEDNAČINE

### Diferencijalne jednačine koje razdvajaju promenljive

- ★  $y' = \frac{y^2}{x^3}$  Rešenje:  $y = \frac{2x^2}{1+2Cx^2}$
- ★  $yy' = xe^{x^2-y^2}$  Rešenje:  $y^2 = \ln(e^{x^2} + C)$
- ★  $(xy^2 + x)dx + (yx^2 + y)dy = 0$  Rešenje:  $y^2 = \frac{C}{x^2+1} - 1$
- ★  $(3 + 2y)dx - (3 - x)dy = 0$  Rešenje:  $(3 - x)\sqrt{3 + 2y} = C$

### Homogene diferencijalne jednačine

- ★  $y' = \frac{2x-3y}{x-2y}$  Rešenje:  $\frac{x}{2(y-x)} = \ln(y-x) + C$
- ★  $\frac{dy}{y(\ln y - \ln x + 1)} = \frac{dx}{x}$  Rešenje:  $y = x e^{xC}$
- ★  $y - x + (y+x)y' = 0$  Rešenje:  $y^2 + 2xy - x^2 = C$
- ★  $xyy' + x^2 - y^2 = 0$  Rešenje:  $y^2 = x^2 \ln \frac{C^2}{x^2}$
- ★  $y' = \frac{2x+3y}{3x+4y}$  Rešenje:  $2y^2 + 3xy + x^2 = C$
- ★  $(y + \sqrt{xy})dx = xdy$  Rešenje:  $x \ln Cx = 2\sqrt{xy}$
- ★  $y' = e^{-\frac{y}{x}} + \frac{y}{x}$  Rešenje:  $y = x \ln \ln Cx$

### Diferencijalne jednačine koje se svode na homogene

- ★  $y' = \frac{2x+y+3}{x+y+2}$  Rešenje:  $\left(\frac{y+1}{x+1} - \sqrt{2}\right)^{\frac{-2-\sqrt{2}}{4}} \left(\frac{y+1}{x+1} + \sqrt{2}\right)^{\frac{-2+\sqrt{2}}{4}} = x+1+C$
- ★  $y' = \frac{6x-3y-18}{x+2y+2}$  Rešenje:  $(y-x+4)^3(y+3x-4)^5 = \frac{1}{2^4 C^8}$
- ★  $(2x+y+1)dx - (4x+2y-3)dy = 0$  Rešenje:  $2x+y-1 = Ce^{2y-x}$
- ★  $y' = \frac{4x-y-1}{y-x-2}$  Rešenje:  $C^4(4x^2 - 8x - y^2 + 6y - 5)(2x+y-5)^2 = 1$
- ★  $(2x+4y+1)dy = (x+2y+3)dx$  Reš:  $8y-5 \ln |4x+8y+7| - 4x = -5 \ln 7$
- ★  $(3x+3y+1)dy + (x+y+1)dx = 0$  Rešenje:  $3y + \ln |x+y| = -x+C$

### Linearna i Bernulijeva diferencijalna jednačina prvog reda

- ★  $y' - \frac{y}{x} = 2x^2 + 5$  Rešenje:  $y = x(x^2 + 5 \ln x + C)$
- ★  $x' + \frac{x}{y^2} = e^{\frac{1}{y}} \cos^2(y+5)$  Rešenje:  $x = e^{\frac{1}{y}} \left(\frac{1}{2}(y+5) + \frac{1}{4} \sin(2y+10)\right) + C$
- ★  $y' + y \cos x = -\sin x \cos x$  Rešenje:  $y = 1 - \sin x + Ce^{-\sin x}$
- ★  $y' - \frac{y}{x} = y^3 \arctg(x^3 + 5)$   
Rešenje:  $-3x^2y = 2(x^3 + 5) \arctg(x^3 + 5) - \ln(1 + (x^3 + 5)^2) + C$
- ★  $y' + \frac{y}{x} = x$  Rešenje:  $y = \frac{1}{x} \left(C + \frac{x^3}{3}\right)$
- ★  $y' - \frac{3y}{x-2} = (x-2)^4$  Rešenje:  $y = (x-2)^3 \left(\frac{(x-2)^2}{2} + C\right)$

- ★  $dy = y \cot x dx - \frac{dx}{\sin x}$  Rešenje:  $y = \cos x + C \sin x$
- ★  $xy' = y + x^3 + 3x^2 - 2x$  Rešenje:  $2y = x^3 + 6x^2 - 4 \ln x + Cx$
- ★  $(1 + y^2)dx = (\arctan y - x)dy$  Rešenje :  $x = \arctan y - 1 + Ce^{\arctan y}$
- ★  $y' + \frac{y}{x} = -xy^2$  Rešenje :  $y = \frac{1}{x(x+C)}$
- ★  $xy' - 4y - x^2\sqrt{y} = 0$  Rešenje :  $y = x^4(C + \frac{1}{2} \ln |x|)^2$
- ★  $ydx + (x - \frac{1}{2}x^3y)dy = 0$  Rešenje :  $x^2y(1 - Cy) = 1$
- ★  $3y' \cos x + y \sin x - \frac{1}{y^2} = 0$  Rešenje :  $y = (\sin x + C \cos x)^{\frac{1}{3}}$
- ★  $(2xy^5 - y)dx + 2xdy = 0$  Rešenje :  $y^4 = \frac{3x^2}{4x^3+C}$

**Diferencijalne jednačine višeg reda (snižavanje reda)**

- ★  $y'' = e^x \sin x$  Rešenje:  $y = -\frac{1}{2}e^x \cos x + \frac{C_1}{2}x + \frac{C_2}{2}$
- ★  $y''' = \ln x$  Rešenje:  $y = \frac{x^3}{6} \ln x - \frac{x^3}{18} - \frac{x^3}{12} + \frac{C_1}{2}x^2 + C_2x + C_3$
- ★  $y^{(IV)} = x^5 + \sin x$  Rešenje:  $y = \frac{x^9}{2988} + \sin x + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3x + C_4$
- ★  $y''x = y'$  Rešenje:  $y = \frac{C_1}{2}x^2 + C_2$
- ★  $y''(y')^3 = 1$  Rešenje:  $y = \frac{5\sqrt{2}}{4}(x + C_1)\sqrt[4]{x + C_1} + \frac{C_2}{4}$
- ★  $y''y' = -x$  Rešenje:  $y = \frac{C_1}{2} \arcsin(\frac{x}{\sqrt{C_1}}) + \frac{C_1}{4} \sin(2 \arcsin(\frac{x}{\sqrt{C_1}})) + C_2$

**Linearna diferencijalna jednačina višeg reda sa konstantnim koeficijentima**

- ★  $y'' + y' - 2y = 2x^3 + x^2 - 2x + 2$   
Rešenje:  $y = C_1e^x + C_2e^{-2x} - x^3 - 2x^2 - 4x - 5$
- ★  $y'' + y' - 2y = 9e^{4x}$  Rešenje:  $y = C_1e^x + C_2e^{-2x} + \frac{1}{2}e^{4x}$
- ★  $y'' + y' - 2y = e^x$  Rešenje:  $y = C_1e^x + C_2e^{-2x} + \frac{1}{3}e^x$
- ★  $y''' - 5y'' + 3y' + 9y = (4x + 11)e^{-x}$   
Rešenje:  $y = C_1e^{-x} + C_2e^{3x} + C_3xe^{3x} + (\frac{1}{8}x + 1)e^{-x}$
- ★  $y''' + 4y'' + 4y' + 16y = \sin(3x) - 7 \cos(3x)$   
Rešenje:  $y = C_1e^{-4x} + C_2 \cos(2x) + C_3 \sin(2x) + \frac{1}{5} \sin(3x) - \frac{1}{5} \cos(3x)$

**Lagranžova metoda varijacije konstanti**

- ★  $y'' - 2y' = e^x \sin x$  Rešenje:  $y = D_1 + D_2e^{2x} - \frac{1}{2}e^x \sin x$
- ★  $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$  Rešenje:  $y = D_1e^{3x} + D_2xe^{3x} - e^{3x} \ln x - e^{3x}$
- ★  $y'' - y' - 2y = e^{2x} \sin 3x$  Reš:  $y = D_1e^{-x} + D_2e^{2x} - \frac{\sin 3x e^{2x}}{82} - \frac{\cos 3x e^{2x}}{738}$
- ★  $y'' + y = \frac{1}{\cos x}$  Reš:  $y = D_1 \cos x + D_2 \sin x + \ln(\cos x) \cos x + x \sin x$

★  $y''' - 3y'' + 3y' - y = \frac{e^x}{x^3}$  Reš:  $y = D_1e^x + D_2xe^x + D_3x^2e^x + \frac{3}{4}e^x + \frac{1}{2}e^x \ln x$

**Rešiti diferencijalne jednačine**

★  $y' = \frac{1}{\cos y + x \tan y}$  Rešenje:  $x = \frac{y}{2 \cos y} + \frac{\sin y}{2} + \frac{C}{\cos y}$

★  $y' = \frac{1}{\sin(\frac{\pi}{2} + \frac{y}{x})} + \frac{y}{x}$  Rešenje:  $x = C_1 e^{\sin \frac{y}{x}}$

★  $y + xy \sin x = \frac{1}{y} y'$  Rešenje:  $C = \frac{1}{y} + x - x \cos x + \sin x$

★  $(x^2 + 2y^2)dx - xydy = 0$  Rešenje:  $x^2 + y^2 = x^4 C_1$

★  $y' = xy(1 - e^x)$  Rešenje:  $\ln y = \frac{x^2}{2} - xe^x + e^x + C$

★  $\frac{dy}{dx} = x^2 \ln x + \frac{1}{x \ln x} y$  Rešenje:  $y = (\frac{x^3}{3} + C) \ln x$

★  $x' = \frac{\ln(\frac{x}{y}) + 1}{\frac{y}{x}}$  Rešenje:  $y = \frac{x}{y} \left( \ln(\frac{x}{y}) - 1 \right) + C$

★  $e^{x+2y} dx + 2^{2x+2y} dy = 0$  Rešenje:  $\frac{(\frac{e}{4})^x}{1-2 \ln 2} + \frac{(\frac{2}{e})^{2y}}{2 \ln 2 - 2} = C$

★  $y' = \cos(2x) \left( \frac{1}{y} + y \right)$  Rešenje:  $y^2 = C e^{\sin(2x)} - 1$

★  $y''' + y'' - y' + 15y = \sin(2x)$

Rešenje:  $y = C_1 e^{-3x} + e^x (C_2 \sin(2x) + C_3 \cos(2x)) + \frac{1}{221} (10 \cos(2x) + 11 \sin(2x))$

★  $y' - y \tan x = -y^2 \cos x$  Rešenje:  $y = \frac{1}{(x+C) \cos x}$

★  $(2x^2 y \ln y - x) y' = y$  Rešenje:  $xy(C - \ln y^2) = 1$

★  $(2x - 4y + 6) dx + (x + y - 3) dy = 0$  Rešenje:  $(y - 2x)^3 = C(y - x - 1)^2$

★  $(x - y - 1) + (y - x + 2) y' = 0$  Rešenje:  $2(x - y) - \frac{(x - y)^2}{2} = x + C$

★  $y'' + 2y = x^2 + 4$  Rešenje:  $y = C_1 \cos(\sqrt{2}x) + C_2 \sin(\sqrt{2}x) + \frac{1}{2}x^2 + \frac{3}{2}$

★  $y''' + y'' = x^2 + 1 + 3e^x$  Reš:  $y = C_1 + xC_2 + C_3 e^{-x} + \frac{1}{12}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 + \frac{3}{2}e^x$

★  $y''' - 5y'' + 8y' - 4y = e^{2x} + e^{3x}, y(0) = 1, y'(0) = 1, y''(0) = 0$

Rešenje:  $y = \frac{1}{2}e^x + \frac{1}{2}e^{2x} - 2xe^{2x} + \frac{1}{2}x^2 e^{2x} + \frac{1}{2}e^{3x}$

**Rešiti diferencijalne jednačine**

★  $y' = \frac{x-y}{x+2y-6}$

★  $y'' + y' + 4y = x^2 + 5x + xe^{3x}$

★  $y'' - 4y' + 4y = \frac{e^{2x}}{x^2}$

★  $y' = \frac{x+y-5}{-x+2y-4}$

★  $y'' - 4y' = x^2 + 4 + 2e^{4x} + e^x$

★  $(x + 2y + 4) dy + (4x - 7y + 1) dx = 0$

★  $dy = \cos x (y + \sin x) dx$

★  $y'' + 2y' + 4y = \sin x + 2 \cos x + x + 3$

- ★  $y''' + 3y'' + y' - 5y = 2x^3 + x^2 + 1 + 3e^x$
- ★  $xydy = (x^2 + 5y^2)dx$
- ★  $y' = x^3(x^4 + 4y)$
- ★  $y''' + y'' - 4y' - 4y = 3\cos x + 2\sin x$
- ★  $y' + y \cos x = \sin x \cos x$
- ★  $(y^2 + 5)dy = (xy + y \sin x + xy \cos x)$
- ★  $y^{VI} - y = 0$
- ★  $y'' - 9y = x^2 - x + 2 - 3e^{-3x} + 2 \cos x$
- ★  $\frac{y'}{4^x 2^{2y}} = 3^y 2^{4x}, y(0) = 0$
- ★  $y' - \frac{2xy}{x^2+1} = x^2$
- ★  $y' + y \sin x = y^2 \sin x \cos x$
- ★  $y'' + 2y' - 3y = 12xe^x$
- ★  $(x^2 + xy)dx = (xy + y^2)dy$
- ★  $y'' + y' - 2y = -2x^2 + 4x - 3, y(0) = 1, y'(0) = 0.$
- ★  $y(2y' - 1) + 5 = 3(x + y') + xy'$
- ★  $y''' + y' = 2e^x + \sin 3x - x^2$
- ★  $y' = \frac{2x-y+3}{x+y}$
- ★  $y^{(IV)} - y = \sin 2x + 2e^{-x}$
- ★  $y'' - y' - 2y = e^{2x} \sin 3x$
- ★  $(x - y)dy = (2x + y - 6)dx$
- ★  $y' - \frac{y}{x} = y^2 x \ln x$
- ★  $y' + y \operatorname{tg} x = \frac{1}{\cos x}$
- ★  $y^{(IV)} - y = e^x + \sin(2x) + x^4 + 1$
- ★  $y'' + 4y - \frac{1}{\sin^2 x} = 0$
- ★  $(2x - 2y)dx = (x + y + 1)dy$
- ★  $y^{(IV)} - 3y'' + 2y = e^x + x^2 - 3$
- ★  $4x + 2x'y = \cos y$
- ★  $(2x + 3y - 5)dx = (x + y - 2)dy$
- ★  $y^{(IV)} - 8y'' - 9y = 4e^{3x}$
- ★  $y' = \frac{2x-3y-5}{3x-4y-7}$
- ★  $y''' + y = x + e^{-2x} + 2003$
- ★  $y'' + 2y' - 3xe^x = 0$
- ★  $xy' + y = y^2 \ln x$

- ★  $y' + \frac{y}{x} = -xy^2$
- ★  $(1 + x^2)dy = (\sqrt{1 + x^2} \sin x - xy)dx$
- ★  $y'' + \frac{y'}{x} = \ln x$
- ★  $y' = \frac{4x+y-5}{-3x-y+3}$
- ★  $y^2 dx - xy dy = \sqrt{x} y^3 dy$
- ★  $y'' - 5y' + 6y = 2x^2 - 2x + 3 + 3e^{3x} + 5 \sin(4x)$
- ★  $y'' + y' = xe^{-x}$
- ★  $2^x 2y + 3^x 3^{-2y} \frac{dy}{dx} = 0$
- ★  $y = (y + x)y' + x$
- ★  $\frac{y'-1}{y'+1} = \frac{x}{y}$
- ★  $(x + y)dx = (x - y)dy$
- ★  $y'' + y' = 24e^{3x} + 10 \cos(2x) + 4x + 3$
- ★  $(yx^2 + 2ye^{2x} \sin x)dx + (-2 \ln y - 2y \cos y + 2y^4)dy = 0$
- ★  $y'' - y' - 2y = 2x^2 + 4x + 1 - 130 \cos(3x)$
- ★  $y'' - 3y' + 2y = \frac{x}{e^x}$
- ★  $y'' + 6y' + 9y = (4x - 3)e^{3x}$
- ★  $y''' - y = e^x + \cos x + \sin(2x)$
- ★  $\frac{2x+3y-5}{x-2y+1} = y'$