Granične vrednosti funkcije
- formule i zadaci -

2010/2011

Graniňa vrednost funkcije

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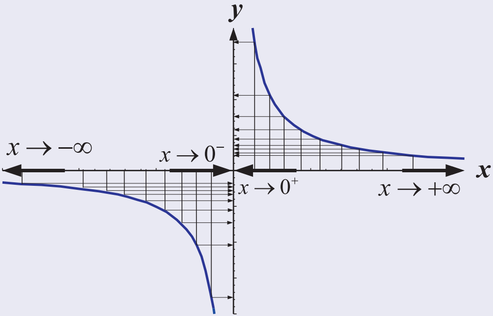
Funkcija: $f(x) = \frac{1}{x}$



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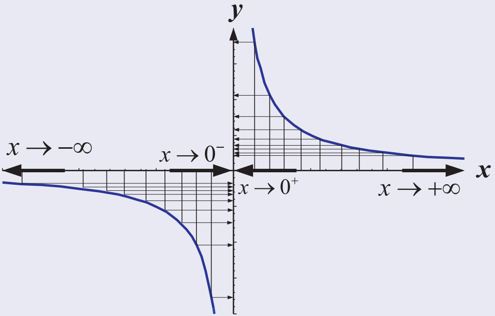

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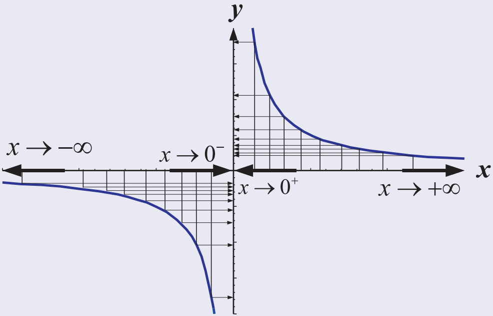

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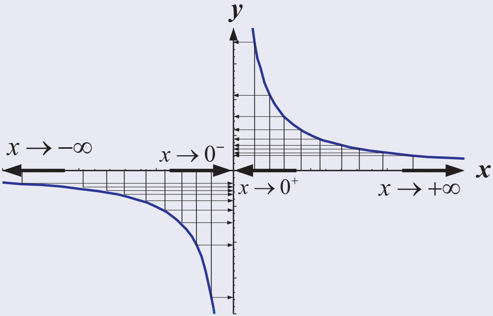

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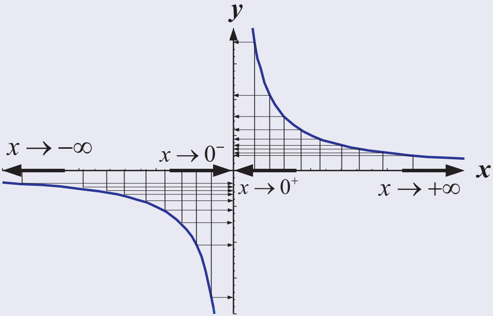

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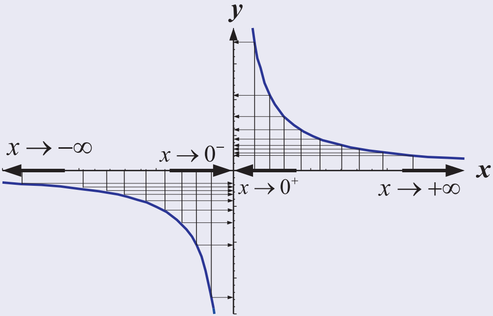

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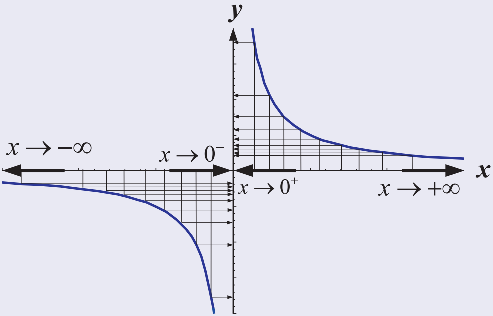

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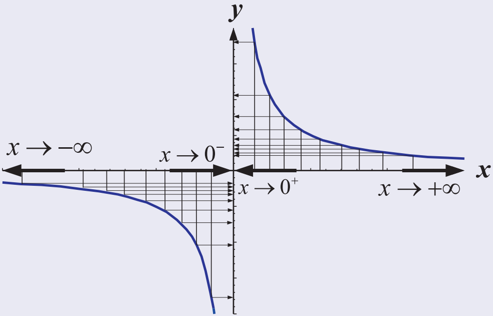

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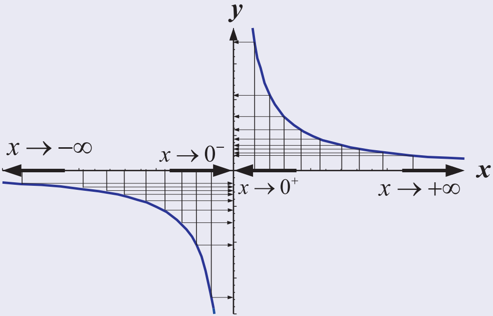

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Zaključak

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$$\lim_{x \to +\infty} \frac{a}{x^{\alpha}} = 0, \quad a \in \mathbb{R}, \quad \alpha > 0$$

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$$\lim_{x \to 0} \frac{1}{x} \quad \text{ne postoji}$$

Neodređeni izrazi

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Postoji 7 neodređenih izraza i to:

$$\frac{0}{0},\ \ \frac{\infty}{\infty},\ \ 0\cdot\infty,\ \ \infty-\infty,\ \ 1^{\infty},\ \ 0^{0},\ \ \infty^{0}$$

Zadatak 1.

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a) $\lim\limits_{x\to+\infty} \dfrac{5x^3+3x^2+3x+5}{3x^3+x^2-x+8} =$

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a) $$\lim_{x \to +\infty} \frac{5x^3 + 3x^2 + 3x + 5}{3x^3 + x^2 - x + 8} = \lim_{x \to +\infty} \frac{x^3 \left(5 + \frac{3}{x} + \frac{3}{x^2} + \frac{5}{x^3}\right)}{x^3 \left(3 + \frac{1}{x} - \frac{1}{x^2} + \frac{8}{x^3}\right)} = \frac{5}{3}$$

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b) $\lim\limits_{x \to +\infty} \frac{6x^4 - 8x^2 + 5}{2x^6 - 4x^2 + 10x - 8} =$

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Izrazi $\frac{\infty}{\infty}$

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c) $$\lim_{x\to-\infty}\frac{x^7+5x^6-7x^2+1}{x^6-3x^5-2x+1}=\lim_{x\to-\infty}\frac{x^6\left(x+5-\frac{7}{x^4}+\frac{1}{x^6}\right)}{x^6\left(1-\frac{3}{x}-\frac{2}{x^5}+\frac{1}{x^6}\right)}=$$

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Izrazi $\infty - \infty$

Zadatak 2.

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$$\lim_{x \to +\infty} \left(\sqrt{x^2 - 2} - \sqrt{x^2 + 2} \right)$$

Zadatak 2.

$$\lim_{x\to+\infty}\left(\sqrt{x^2-2}-\sqrt{x^2+2}\right)$$

$$=\lim_{x\to+\infty}\left(\sqrt{x^2-2}-\sqrt{x^2+2}\right)\cdot\frac{\sqrt{x^2-2}+\sqrt{x^2+2}}{\sqrt{x^2-2}+\sqrt{x^2+2}}$$

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$$= \lim_{x\to+\infty}\frac{(x^2-2)-(x^2+2)}{\sqrt{x^2-2}+\sqrt{x^2+2}}$$

$$= \lim_{x\to+\infty}\frac{-4}{\sqrt{x^2-2}+\sqrt{x^2+2}} = 0$$

Zadatak 3.

Izrazi $\infty - \infty$

Zadatak 3.

$$\lim_{x \to +\infty} \left(\frac{x^4 + 2x^3}{1 + x^3} - x \right)$$

Zadatak 3.

$$\lim_{x \to +\infty} \left(\frac{x^4 + 2x^3}{1 + x^3} - x \right) = \lim_{x \to +\infty} \frac{x^4 + 2x^3 - x\left(1 + x^3\right)}{1 + x^3}$$

Zadatak 3.

$$\lim_{x\to+\infty}\left(\frac{x^4+2x^3}{1+x^3}-x\right) = \lim_{x\to+\infty}\frac{x^4+2x^3-x\left(1+x^3\right)}{1+x^3}$$

$$= \lim_{x\to+\infty}\frac{2x^3-x}{1+x^3}$$

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$$\lim_{x\to+\infty}\left(\frac{x^4+2x^3}{1+x^3}-x\right)=\lim_{x\to+\infty}\frac{x^4+2x^3-x\left(1+x^3\right)}{1+x^3}$$

$$=\lim_{x\to+\infty}\frac{2x^3-x}{1+x^3}=\lim_{x\to+\infty}\frac{x^3\left(2-\frac{1}{x^2}\right)}{x^3\left(1+\frac{1}{x^3}\right)}=2$$

Zadatak 4.

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$$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16}$$

Zadatak 4.

$$\lim_{x\to 16}\frac{\sqrt{x}-4}{x-16}=\lim_{x\to 16}\frac{\sqrt{x}-4}{(\sqrt{x})^2-4^2}$$

Izrazi $\frac{0}{0}$

Zadatak 4.

$$\lim_{x\to 16}\frac{\sqrt{x}-4}{x-16}=\lim_{x\to 16}\frac{\sqrt{x}-4}{(\sqrt{x})^2-4^2}$$

$$=\lim_{x\to 16}\frac{\sqrt{x}-4}{(\sqrt{x}-4)(\sqrt{x}+4)}$$

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$$=\lim_{x\to 16}\frac{\sqrt{x}-4}{(\sqrt{x}-4)(\sqrt{x}+4)}=\lim_{x\to 16}\frac{1}{\sqrt{x}+4}=\frac{1}{8}$$

Zadatak 5.

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$$\lim_{x\to 2} \frac{\sqrt{3-2x+x^2}-\sqrt{x^2-x+1}}{2x-x^2}$$

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$$\lim_{x\to 2}\frac{\sqrt{3-2x+x^2}-\sqrt{x^2-x+1}}{2x-x^2}$$

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Zadatak 5.

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$$=\lim_{x\to 2}\frac{3-2x+x^2-(x^2-x+1)}{(2x-x^2)\left(\sqrt{3-2x+x^2}+\sqrt{x^2-x+1}\right)}$$

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$$=\lim_{x\to 2}\frac{2-x}{x(2-x)\left(\sqrt{3-2x+x^2}+\sqrt{x^2-x+1}\right)}$$

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$$= \lim_{x\to 2} \frac{2-x}{x(2-x)\left(\sqrt{3-2x+x^2}+\sqrt{x^2-x+1}\right)}$$

$$= \lim_{x\to 2} \frac{1}{x\left(\sqrt{3-2x+x^2}+\sqrt{x^2-x+1}\right)} = \frac{1}{4\sqrt{3}} = \frac{\sqrt{3}}{12}$$

Zadatak 6.

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$$\lim_{x \to 0} \frac{4x}{\sqrt[3]{x+27} - 3}$$

Zadatak 6.

$$\lim_{x\to 0} \frac{4x}{\sqrt[3]{x+27}-3} = \lim_{x\to 0} \frac{4x}{\sqrt[3]{x+27}-3} \cdot \frac{\left(\sqrt[3]{x+27}\right)^2 + 3\sqrt[3]{x+27}+9}{\left(\sqrt[3]{x+27}\right)^2 + 3\sqrt[3]{x+27}+9}$$

Zadatak 6.

$$\lim_{x\to 0}\frac{4x}{\sqrt[3]{x+27}-3}=\lim_{x\to 0}\frac{4x}{\sqrt[3]{x+27}-3}\cdot\frac{\left(\sqrt[3]{x+27}\right)^2+3\sqrt[3]{x+27}+9}{\left(\sqrt[3]{x+27}\right)^2+3\sqrt[3]{x+27}+9}$$

$$=\lim_{x\to 0}\frac{4x\left(\left(\sqrt[3]{x+27}\right)^2+3\sqrt[3]{x+27}+9\right)}{x+27-27}$$

Zadatak 6.

$$\lim_{x\to 0}\frac{4x}{\sqrt[3]{x+27}-3}=\lim_{x\to 0}\frac{4x}{\sqrt[3]{x+27}-3}\cdot\frac{\left(\sqrt[3]{x+27}\right)^2+3\sqrt[3]{x+27}+9}{\left(\sqrt[3]{x+27}\right)^2+3\sqrt[3]{x+27}+9}$$

$$=\lim_{x\to 0}\frac{4x\left(\left(\sqrt[3]{x+27}\right)^2+3\sqrt[3]{x+27}+9\right)}{x+27-27}$$

$$=\lim_{x\to 0}\frac{4\left(\left(\sqrt[3]{x+27}\right)^2+3\sqrt[3]{x+27}+9\right)}{1}=108$$

Zadatak 7.

Zadatak 7.

$$\lim_{x \to 1} \left(\frac{3}{1 - x^3} + \frac{1}{x - 1} \right)$$

Zadatak 7.

$$\lim_{x\to 1}\left(\frac{3}{1-x^3}+\frac{1}{x-1}\right)=\lim_{x\to 1}\left(\frac{3}{(1-x)(1+x+x^2)}+\frac{1}{x-1}\right)$$

Izrazi $\frac{0}{0}$

Zadatak 7.

$$\lim_{x\to 1}\left(\frac{3}{1-x^3}+\frac{1}{x-1}\right)=\lim_{x\to 1}\left(\frac{3}{(1-x)(1+x+x^2)}+\frac{1}{x-1}\right)$$

$$=\lim_{x\to 1}\frac{3-(1+x+x^2)}{(1-x)(1+x+x^2)}$$

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$$=\lim_{x\to 1}\frac{3-(1+x+x^2)}{(1-x)(1+x+x^2)}=\lim_{x\to 1}\frac{2-x-x^2}{(1-x)(1+x+x^2)}$$

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$$\lim_{x\to 1}\left(\frac{3}{1-x^3}+\frac{1}{x-1}\right)=\lim_{x\to 1}\left(\frac{3}{(1-x)(1+x+x^2)}+\frac{1}{x-1}\right)$$

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$$=\lim_{x\to 1}\frac{(1-x)(2+x)}{(1-x)(1+x+x^2)}$$

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$$=\lim_{x\to 1}\frac{(1-x)(2+x)}{(1-x)(1+x+x^2)}=\lim_{x\to 1}\frac{2+x}{1+x+x^2}=1$$

Poznate granične vrednosti

Poznate granične vrednosti

$$\lim_{x\to 0}\frac{\sin x}{x}=1,\qquad \lim_{x\to x_0}\frac{\sin f(x)}{f(x)}=1 \ \text{ gde je } \ \lim_{x\to x_0} f(x)=0$$

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$$\lim_{x\to 0}\frac{e^x-1}{x}=1,\qquad \lim_{x\to x_0}\frac{e^{f(x)}-1}{f(x)}=1 \;\text{ gde je }\; \lim_{x\to x_0} f(x)=0$$

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$$\lim_{x\to 0}\frac{e^x-1}{x}=1,\qquad \lim_{x\to x_0}\frac{e^{f(x)}-1}{f(x)}=1\ \text{ gde je }\ \lim_{x\to x_0}f(x)=0$$

$$\lim_{x\to 0}\frac{\ln(x+1)}{x}=1,\qquad \lim_{x\to x_0}\frac{\ln(f(x)+1)}{f(x)}=1\ \text{ gde je }\ \lim_{x\to x_0}f(x)=0$$

$$\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x=e,\qquad \lim_{x\to\pm\infty}\left(1-\frac{1}{x}\right)^x=\frac{1}{e}$$

Granična vrednost $\lim_{x \to 0} \frac{\sin x}{x} = 1$

Zadatak 8.

a) $\lim_{x \to 0} \frac{\sin 4x}{\sin 5x}$,

b) $\lim_{x \to 0} \frac{\operatorname{tg} 2x}{2x}$,

c) $\lim_{x \to 0} \left(\frac{2}{\sin 2x \cdot \sin x} - \frac{1}{\sin^2 x} \right)$,

d) $\lim_{x \to 0} \frac{\sin x}{\sin 5x - \sin 7x}$,

e) $\lim_{x \to 0} (3x \cdot \operatorname{ctg} 3x)$,

f) $\lim_{x \to 0} \frac{4 \cdot \sin(\sqrt{x+1} - 1)}{x}$,

g) $\lim_{x \to 0} \frac{x^2}{\sqrt{1 + x \sin x} - \sqrt{\cos x}}$,

h) $\lim_{x \to 2} \frac{\sin(x-2)}{8 - x^3}$,

i) $\lim_{x \to -1} \frac{\sin(x+1)}{x^2 - 1}$.

Granična vrednost $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$, $\lim_{x\to 0} \frac{\ln(x+1)}{x} = 1$

Zadatak 9.

a) $\lim_{x\to 0} \frac{e^{7x} - 1}{2x}$, b) $\lim_{x\to 0} \frac{e^{3x} - 1}{\sin 4x}$,

c) $\lim_{x\to 0} \frac{\ln(1+3x)}{x}$, d) $\lim_{x\to 4} \frac{\log_4 x - 1}{x^3 - 64}$.

Granična vrednost

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x = e, \qquad \lim_{x\to+\infty}\left(1-\frac{1}{x}\right)^x = e^{-1}$$

Zadatak 10.

a) $\lim\limits_{x\to-\infty}\left(1+\dfrac{1}{x}\right)^x$, b) $\lim\limits_{x\to-\infty}\left(1-\dfrac{1}{x}\right)^x$, c) $\lim\limits_{x\to+\infty}\left(\dfrac{x+2}{x-3}\right)^x$,

d) $\lim\limits_{x\to 0}(1+x)^{1/x}$, e) $\lim\limits_{x\to 0}(1-x)^{1/x}$, f) $\lim\limits_{x\to \pi/2}(1+\operatorname{ctg} x)^{\operatorname{tg} x}$.