NEODREDJENI INTEGRAL

1. **Rešiti sledeće integrale:**

a) $\int (x^3 - 2x^2 + 2x - 1)\, dx$ b) $\int (x^2 - 1)^3\, dx$ c) $\int (1 + \sqrt{x})^4\, dx$

d) $\int \dfrac{2x^3 + 3x^2 - 4x - 3}{2x^2}\, dx$ e) $\int \dfrac{(x-1)(x^2-3)}{3x^2}dx$ f) $\int \dfrac{x-1}{\sqrt{x^3}}\, dx$

g) $\int \dfrac{4x^4 + 5\sqrt[3]{x^7} + 3\sqrt[4]{x^3} + \sqrt{x} + 1}{\sqrt{x}}\, dx$ h) $\int \sqrt{x\sqrt{x\sqrt{x}}}\, dx$

i) $\int (\cos x + 4e^x - x^3)\, dx$ j) $\int e^x (1 - \dfrac{e^{-x}}{x^2})\, dx$ k) $\int a^x (1 + \dfrac{a^{-x}}{\sqrt{x^3}})\, dx$

l) $\int \dfrac{dx}{\sqrt{3 - 3x^2}}$ m) $\int (\dfrac{3}{1+x^2} - \dfrac{2}{\sqrt{1-x^2}})\, dx$ n) $\int \dfrac{1 + 2x^2}{x^2(1+x^2)}\, dx$

o) $\int \dfrac{(1+x)^2}{x(1+x^2)}\, dx$ p) $\int \dfrac{x^4}{1+x^2}\, dx$

Rešenja 1 :

a) $\dfrac{x^4}{4} - \dfrac{2}{3}x^3 + x^2 - x + C$ b) $\dfrac{x^7}{7} - \dfrac{3}{5}x^5 + x^3 - x + C$ c) $x + \dfrac{8}{3}x^{\frac{3}{2}} + 3x^2 + \dfrac{8}{5}x^{\frac{5}{2}} + \dfrac{x^3}{3} + C$

d) $\dfrac{x^2}{2} + \dfrac{3x}{2} - 2\ln x + \dfrac{3}{2x} + C$ e) $\dfrac{x^2}{6} - \dfrac{x}{3} - \ln x - \dfrac{1}{x} + C$ f) $\dfrac{2}{\sqrt{x}}(x+1) + C$

g) $\dfrac{8}{9}\sqrt{x^9} + \dfrac{30}{17}\sqrt[6]{x^{17}} + \dfrac{12}{5}\sqrt[4]{x^5} + x + 2\sqrt{x} + C$ h) $\dfrac{8}{15}x^{\frac{15}{8}} + C$

i) $\sin x + 4e^x - \dfrac{x^4}{4} + C$ j) $e^x + \dfrac{1}{x} + C$ k) $\dfrac{a^x}{\ln a} - \dfrac{2}{\sqrt{x}} + C$

l) $\dfrac{1}{\sqrt{3}}\arcsin x + C$ m) $3arctgx - 2\arcsin x + C$ n) $arctgx - \dfrac{1}{x} + C$

o) $\ln x + 2arctgx + C$ p) $\dfrac{x^3}{3} - x + arctgx + C$

2. **Odrediti integrale :**

a) $\int \dfrac{1}{ax+b}\, dx$ b) $\int \dfrac{6}{(2x+4)^2}\, dx$ c) $\int \dfrac{2x+6}{x^2+6x-3}\, dx$

d) $\int \dfrac{x^4}{\sqrt{4+x^5}}\, dx$ e) $\int \dfrac{x}{\sqrt{x+5}}\, dx$ f) $\int e^{-3x}\, dx$

g) $\int \dfrac{e^{\sqrt{x}+3}}{\sqrt{x}}\, dx$ h) $\int e^{x^2+x+5}(2x+1)\, dx$ i) $\int \dfrac{1}{x\ln x}\, dx$

j) $\int x\sin(1-x^2)\, dx$ k) $\int \dfrac{x}{\cos^2 x^2}\, dx$ l) $\int \dfrac{arctg^2 x}{1+x^2}\, dx$

m) $\int \dfrac{1}{\sqrt{1-x^2}\,\arcsin x}\,dx$　　　　n) $\int \sqrt{1+4\sin x}\,\cos x\,dx$　　　　o) $\int \dfrac{2x-\sqrt{\arcsin x}}{\sqrt{1-x^2}}\,dx$

p) $\int \dfrac{1}{\sqrt{1+x}-\sqrt{x-1}}\,dx$

Rešenja 2 :

a) $\dfrac{1}{a}\ln(ax+b)+C$

b) $-\dfrac{3}{2x+4}+C$

c) $\ln(x^2+6x-3)+C$

d) $\dfrac{2}{5}\sqrt{4+x^5}+C$

e) $\dfrac{2}{3}\sqrt{(x+5)^3}-10\sqrt{x+5}+C$

f) $-\dfrac{e^{-3x}}{3}+C$

g) $2e^{\sqrt{x}+3}+C$

h) $e^{x^2+x+5}+C$

i) $\ln(\ln x)+C$

j) $\dfrac{1}{2}\cos(1-x^2)+C$

k) $\dfrac{1}{2}tgx^2+C$

l) $\dfrac{1}{3}(arctgx)^3+C$

m) $\ln(\arcsin x)+C$

n) $\dfrac{1}{6}(1+4\sin x)^{\frac{3}{2}}+C$

o) $C-2\sqrt{1-x^2}-\dfrac{2}{3}\sqrt{(\arcsin x)^3}$

p) $\dfrac{1}{3}\left(\sqrt{(x+1)^3}-\sqrt{(x-1)^3}\right)+C$

3. **Rešiti integrale:**

a) $\int x\cos x\,dx$

b) $\int x\sin 2x\,dx$

c) $\int x3^x\,dx$

d) $\int xe^{-x}\,dx$

e) $\int x^2 e^x\,dx$

f) $\int (x^2+5x+6)\cos x\,dx$

g) $\int \dfrac{x}{\cos^2 x}\,dx$

h) $\int xarctgx\,dx$

i) $\int arctgx\,dx$

j) $\int x^a\ln x\,dx$

k) $\int (4x+8)\ln(x+2)\,dx$

l) $\int e^x\sin x\,dx$

m) $\int \cos(\ln x)\,dx$

Rešenja 3 :

a) $x\sin x+\cos x+C$

b) $-\dfrac{x\cos 2x}{2}+\dfrac{1}{4}\sin 2x+C$

c) $\dfrac{x3^x}{\ln 3}-\dfrac{3^x}{\ln^2 3}+C$

d) $-e^{-x}(x+1)+C$

e) $e^x(x^2-2x+2)+C$

f) $(x^2+5x+4)\sin x+(2x+5)\cos x+C$

g) $xtgx+\ln(\cos x)+C$

h) $\dfrac{x^2}{2}arctgx-\dfrac{x}{2}+\dfrac{1}{2}arctgx+C$

i) $xarctgx-\dfrac{1}{2}\ln(x^2+1)+C$

j) $\dfrac{x^{a+1}}{a+1}\ln x-\dfrac{x^{a+1}}{(a+1)^2}+C$

k) $(x+2)^2(2\ln(x+2)-1)+C$

l) $\dfrac{1}{2}e^x(\sin x-\cos x)+C$

m) $\dfrac{x}{2}(\sin(\ln x)+\cos(\ln x))+C$

4. **Odrediti integrale racionalnih funkcija:**

a) $\int \dfrac{1}{(x+a)(x+b)} \, dx$

b) $\int \dfrac{x-5}{x^2-x-2} \, dx$

c) $\int \dfrac{x}{x^4-4x^2+3} \, dx$

d) $\int \dfrac{1}{2x^2-5x+7} \, dx$

e) $\int \dfrac{1}{3x^2-x+1} \, dx$

f) $\int \dfrac{3x-2}{x^2-4x+5} \, dx$

g) $\int \dfrac{(x-1)^2}{x^2+3x+4} \, dx$

h) $\int \dfrac{x^3}{x^2+1} \, dx$

i) $\int \dfrac{x^3+1}{x^3-x^2} \, dx$

j) $\int \dfrac{4x^2+4x-11}{(2x-1)(2x+3)(2x-5)} \, dx$

k) $\int \dfrac{1}{x^3-1} \, dx$

l) $\int \dfrac{x^4}{x^4-1} \, dx$

m) $\int \dfrac{x}{(x-1)(x+1)^2} \, dx$

n) $\int \dfrac{3x-2}{x^2+4x+12} \, dx$

o) $\int \dfrac{8x+4}{x^2+x+5} \, dx$

p) $\int \dfrac{4x+3}{4x^2+6x+2} \, dx$

q) $\int \dfrac{x^3+9x^2-2x-2}{x^2(x^2-1)} \, dx$

r) $\int \dfrac{-3x^3+7x^2-3x+1}{x^2(x-1)^2} \, dx$

s) $\int \dfrac{6x^3+4x^2+4x+3}{x^2(x^2+1)} \, dx$

t) $\int \dfrac{x^3+2}{x^2+1} \, dx$

u) $\int \dfrac{x^2+1}{x^2+5} \, dx$

v) $\int \dfrac{3x^4+2x^3-2x^2+5x-6}{2x-1} \, dx$

w) $\int \dfrac{10x^3+71x^2+155x+100}{x^4+10x^3+35x^2+50x+24} \, dx$

Rešenja 4 :

a) $\dfrac{1}{a-b}\ln(\dfrac{x+b}{x+a})+C$

b) $\ln\dfrac{(x+1)^2}{x-2}+C$

c) $\dfrac{1}{4}\ln\dfrac{x^2-3}{x^2-1}+C$

d) $\dfrac{2}{\sqrt{31}}arctg\dfrac{4x-5}{\sqrt{31}}+C$

e) $\dfrac{2}{\sqrt{11}}arctg\dfrac{6x-1}{\sqrt{11}}+C$

f) $\dfrac{3}{2}\ln(x^2-4x+5)+4arctg(x-2)+C$

g) $x-\dfrac{5}{2}\ln(x^2+3x+4)+\dfrac{9}{\sqrt{7}}arctg\dfrac{2x+3}{\sqrt{7}}+C$

h) $\dfrac{x^2}{2}-\dfrac{1}{2}\ln(x^2+1)+C$

i) $x+\dfrac{1}{x}+\ln\dfrac{(x-1)^2}{x}+C$

j) $\dfrac{1}{8}\ln\dfrac{(2x-1)^2(2x-5)^3}{2x+3}+C$

k) $\dfrac{1}{3}\ln(x-1)-\dfrac{1}{6}\ln(x^2+x+1)-\dfrac{1}{\sqrt{3}}arctg\dfrac{2x+1}{\sqrt{3}}+C$

l) $x+\dfrac{1}{4}\ln\dfrac{x-1}{x+1}-\dfrac{1}{2}arctgx+C$

m) $\dfrac{1}{4}\ln\dfrac{x-1}{x+1}-\dfrac{1}{2(x+1)}+C$

n) $\dfrac{3}{2}\ln(x^2+4x+12)-2\sqrt{2}arctg\Big(\dfrac{x+2}{2\sqrt{2}}\Big)+C$

o) $4\ln(x^2+x+5)+C$

p) $\dfrac{1}{2}\ln(4x^2+6x+2)+C$

q) $\ln\dfrac{(x^2(x-1)^3}{(x+1)^4)}-\dfrac{2}{x}+C$

r) $\ln\Big(\dfrac{1}{x(x-1)^2}\Big)-\dfrac{1}{x}-\dfrac{2}{x-1}+C$

s) $\ln(x^4(x^2+1))+arctgx-\dfrac{3}{x}+C$

t) $2arctgx+\dfrac{1}{2}(x^2+1)-\dfrac{1}{2}\ln(x^2+1)+C$

u) $x-\dfrac{4\sqrt{5}}{5}arctg\Big(\dfrac{x}{\sqrt{5}}\Big)+C$

v) $\dfrac{3}{8}x^4 + \dfrac{7}{12}x^3 - \dfrac{1}{16}x^2 + \dfrac{39}{16}x - \dfrac{57}{32}\ln(2x-1) + C$ 　　　　w) $\ln((x+1)(x+2)^3(x+3)^2(x+4)^4) + C$

5. **Odrediti integrale:**

a) $\displaystyle\int \dfrac{1 - \sin^3 x}{\sin^2 x}\,dx$ 　　　b) $\displaystyle\int 2\sin^2 \dfrac{x}{2}\,dx$ 　　　c) $\displaystyle\int ctg^2 x\,dx$

d) $\displaystyle\int \dfrac{\cos 2x}{\cos^2 x \sin^2 x}\,dx$ 　　　e) $\displaystyle\int \dfrac{1 + \cos^2 x}{1 + \cos 2x}\,dx$ 　　　f) $\displaystyle\int \dfrac{1}{\sin^2 x \cos^2 x}\,dx$

g) $\displaystyle\int \dfrac{\sin x}{\cos^2 x}\,dx$ 　　　h) $\displaystyle\int \dfrac{1 + \sin 2x}{\sin^2 x}\,dx$ 　　　i) $\displaystyle\int \sin^3 x\,dx$

j) $\displaystyle\int (4 + \sin x)^3\,dx$ 　　　k) $\displaystyle\int \cos^3(\sin x)\cos x\,dx$ 　　　l) $\displaystyle\int \sin^2 x \cos^3 x\,dx$

m) $\displaystyle\int \sqrt[12]{\sin^5 x}\cos x\,dx$ 　　　n) $\displaystyle\int \cos^2(2x+1)\,dx$ 　　　o) $\displaystyle\int 2x^3 \sin^2(x^4 - 1)\,dx$

p) $\displaystyle\int \cos^2(x-1)\sin^2(x-1)\,dx$ 　　　q) $\displaystyle\int tg^3 x\,dx$

Rešenja 5 :

a) $\cos x - ctg x + C$ 　　　b) $x - \sin x + C$ 　　　c) $-ctg x - x + C$

d) $-ctg x - tg x + C$ 　　　e) $\dfrac{1}{2}(tg x + x) + C$ 　　　f) $tg x - ctg x + C$

g) $\dfrac{1}{\cos x} + C$ 　　　h) $2\ln(\sin x) - ctg x + C$ 　　　i) $-\cos x + \dfrac{1}{3}\cos^3 x + C$

j) $70x - 49\cos x - 3\sin(2x) + \dfrac{1}{3}\cos^3 x + C$ 　　　k) $\sin(\sin x) - \dfrac{\sin^3(\sin x)}{3} + C$

l) $\dfrac{\sin^3 x}{3} - \dfrac{\sin^5 x}{5} + C$ 　　　m) $\dfrac{12}{17}\sqrt[12]{\sin^{17} x} + C$ 　　　n) $\dfrac{1}{4}(2x+1) + \dfrac{1}{8}\sin(4x+2) + C$

o) $\dfrac{1}{4}(x^4 - 1) - \dfrac{1}{8}\sin(2x^4 - 2) + C$ 　　　p) $\dfrac{1}{8}(x-1) + \dfrac{1}{32}\sin(4x-4) + C$ 　　　q) $\dfrac{1}{2\cos^2 x} + \ln(\cos x) + C$

6. **Odrediti integrale:**

a) $\displaystyle\int \dfrac{3x}{2x+1}\,dx$ 　　　b) $\displaystyle\int \dfrac{ax+b}{cx+d}\,dx$ 　　　c) $\displaystyle\int \dfrac{6x^2}{2x^2+4}\,dx$

d) $\displaystyle\int \dfrac{1}{a^2 + b^2 x^2}\,dx$ 　　　e) $\displaystyle\int \dfrac{x}{4 + x^4}\,dx$ 　　　f) $\displaystyle\int \dfrac{1}{1 + \sin x}\,dx$

g) $\displaystyle\int \dfrac{1}{e^{-x} + e^x}\,dx$ 　　　h) $\displaystyle\int \dfrac{e^{2x}}{\sqrt{e^x + 1}}\,dx$ 　　　i) $\displaystyle\int \dfrac{\ln(x+2)}{4x+8}\,dx$

j) $\displaystyle\int \ln\sqrt{x}\,dx$ 　　　k) $\displaystyle\int e^{\cos x}\sin(\cos x)\sin x\,dx$ 　　　l) $\displaystyle\int (3x^2 + 5x - 7)\cos(2x+1)\,dx$

m) $\displaystyle\int (x^2 + 2)arctg(x-3)\,dx$ 　　　n) $\displaystyle\int \arcsin^2 x\,dx$ 　　　o) $\displaystyle\int \dfrac{arctg\,e^x}{e^x}\,dx$

p) $\int \sqrt{4-x^2}\, dx$
q) $\int \sqrt{12-(x-5)^2}\, dx$
r) $\int x\sqrt{4-x^2}\, dx$

s) $\int x^2\sqrt{4-x^2}\, dx$

Rešenja 6 :

a) $\dfrac{3}{2}x - \dfrac{3}{4}\ln(2x+1) + C$

b) $\dfrac{a}{c}x + \dfrac{bc-ad}{c^2}\ln(cx+d) + C$

c) $3x - 3\sqrt{2}\,arctg\left(\dfrac{x}{\sqrt{2}}\right) + C$

d) $\dfrac{1}{ab}arctg\dfrac{bx}{a} + C$

e) $\dfrac{1}{4}arctg\dfrac{x^2}{2} + C$

f) $\dfrac{\sin x - 1}{\cos x} + C$

g) $arctg\,e^x + C$

h) $\dfrac{2}{3}(e^x - 2)\sqrt{e^x+1} + C$

i) $\dfrac{1}{8}\ln^2(x+2) + C$

j) $x\ln\sqrt{x} - \dfrac{x}{2} + C$

k) $\dfrac{1}{2}e^{\cos x}(\cos(\cos x) - \sin(\cos x)) + C$

l) $\dfrac{6x^2 + 10x - 17}{4}\sin(2x+1) + \dfrac{6x+5}{4}\cos(2x+1) + C$

m) $\left(\dfrac{x^3}{3} + 2x - 12\right)arctg(x-3) - \dfrac{16}{3}\ln(x^2 - 6x + 10) - \dfrac{x^2}{6} - 2x + C$

n) $x\arcsin^2 x + 2\sqrt{1-x^2}\arcsin x - 2x + C$

o) $-e^{-x}arctg\,e^x + x - \dfrac{1}{2}\ln(1+e^{2x}) + C$

p) $2\arcsin\dfrac{x}{2} + \sin(2\arcsin\dfrac{x}{2}) + C$

q) $6\arcsin\left(\dfrac{x-5}{\sqrt{12}}\right) + 3\sin\left(2\arcsin\left(\dfrac{x-5}{\sqrt{12}}\right)\right) + C$

r) $-\dfrac{1}{3}\sqrt{(4-x^2)^3} + C$

s) $2\arcsin\dfrac{x}{2} - \dfrac{1}{2}\sin(4\arcsin\dfrac{x}{2}) + C$